

Unconventional low-energy SUSY from warped geometry

J.A. Casas^{1,2*}, J.R. Espinosa^{2,3†} and I. Navarro^{1‡}

¹ I.E.M. (CSIC), Serrano 123, 28006 Madrid, Spain.

² I.F.T. C-XVI, U.A.M., 28049 Madrid, Spain

³ I.M.A.F.F. (CSIC), Serrano 113 bis, 28006 Madrid, Spain

Abstract

Supersymmetric models with a warped fifth spatial dimension can solve the hierarchy problem, avoiding some shortcomings of non-supersymmetric constructions, and predict a plethora of new phenomena at typical scales Λ not far from the electroweak scale ($\Lambda \sim$ a few TeV). In this paper we derive the low-energy effective theories of these models, valid at energies below Λ . We find that, in general, such effective theories can deviate significantly from the Minimal Supersymmetric Standard Model (MSSM) or other popular extensions of it, like the NMSSM: they have non-minimal Kähler potentials (even in the $M_p \rightarrow \infty$ limit), and the radion is coupled to the visible fields, both in the superpotential and the Kähler potential, in a non-trivial (and quite model-independent) fashion. The corresponding phenomenology is pretty unconventional, in particular the electroweak breaking occurs in a non-radiative way, with $\tan \beta \simeq 1$ as a quite robust prediction, while the mass of the lightest Higgs boson can be as high as ~ 700 GeV.

September 2001

*E-mail: alberto@makoki.iem.csic.es

†E-mail: espinosa@makoki.iem.csic.es

‡E-mail: ignacio@makoki.iem.csic.es

1 Introduction

In the last two years there has been a lot of interest in warped compactifications [1,2] as an alternative mechanism to address the hierarchy problem. In its simplest realization, the Randall-Sundrum (RS) model [1], the action is 5D gravity realized on $\mathcal{M}_4 \times S_1/Z_2$ with negative cosmological constant and two constant three-brane energy-densities located at the Z_2 fixed points. For an appropriate tuning of the latter with the bulk cosmological constant, the metric admits the RS solution¹

$$ds^2 = e^{-2k|x^5|} \eta_{\mu\nu} dx^\mu dx^\nu - (dx^5)^2. \quad (1.1)$$

Here $x^5 \equiv \vartheta r_0$, where the angle ϑ ($-\pi \leq \vartheta \leq \pi$) parametrizes the extra dimension, with ϑ and $-\vartheta$ identified; r_0 is a free parameter giving the radius of the compact S_1 and $k = \mathcal{O}(M_5)$, with M_5 the 5D Planck mass (the 4D Planck mass is $M_p^2 \simeq M_5^3/k$). The branes are located at $x^5 = 0$ (“hidden brane”) and $x^5 = \pi r_0$ (“visible brane”). In the original RS model, all the visible fields (except gravity) are located at the visible brane, and this will be also our assumption here. Then, any mass parameter, m_0 , at the visible brane gets effectively suppressed by the warp factor giving $m = e^{-kr_0\pi} m_0$ [1]. For moderate values of r_0 , the mass hierarchy achieved is of the right order, so that $m \sim \mathcal{O}(\text{TeV})$ for the natural choice $m_0 \sim M_p$. This is the simplest and probably most attractive scenario, although there are variations in the literature.

At first sight, it may seem unnecessary to supersymmetrize these warped constructions. After all, low-energy supersymmetry (SUSY) is an alternative extension of the Standard Model whose primary motivation is also to face the hierarchy problem. However, SUSY may prove very helpful to avoid some shortcomings of the original RS scenario. In particular, it can provide an explanation (at least a partial one) for the correlations between the brane tensions and the bulk cosmological constant [3]. Moreover, SUSY may help to protect the hierarchy against destabilization by radiative corrections. Finally, SUSY is likely a necessary ingredient to make contact with warped superstring constructions. As a result, a lot of effort has been dedicated to supersymmetrize warped scenarios [3–7]. The stabilization of these constructions, and in particular the value of r_0 , is generically connected to the mechanism of SUSY breaking.

Still, from the point of view of low-energy SUSY, one may wonder what is gained by

¹We use the metric signature $(+ - - -)$.

promoting any ordinary version of the MSSM to a higher dimensional warped world. A partial answer is that the warped version has a solution to the μ -problem built in. As any other mass scale in the visible brane, the value of μ is affected by the warp suppression factor, so that μ is naturally \mathcal{O} (TeV). However, a more complete answer to this question requires the phenomenology of the supersymmetric warped worlds to be worked out.

This is the main goal of this paper: to derive the low-energy effective theories of supersymmetric warped constructions and to extract from their the low-energy phenomenology. We will show that, indeed, these effective theories (and the phenomenology they imply) are non-conventional. This might be useful to alleviate some of the drawbacks of the MSSM. We will show in particular that the upper bound on the Higgs mass, which is starting to be worrisome in the conventional MSSM, evaporates. On the other hand, we will see that a non-conventional phenomenology provides many useful tests to distinguish a warped SUSY world from an ordinary MSSM (or further extensions, as the NMSSM). Conversely, the present experimental information imposes non-trivial constraints on supersymmetric warped constructions.

In section 2 we briefly review the construction of SUSY warped scenarios and the extraction of the 4D theory by matching to the 5D complete theory. In the process, we fix some notation and make explicit our starting assumptions (in particular for the choice of Kähler potential). In section 3 we derive the low-energy effective theory in the $M_p \rightarrow \infty$ limit, obtaining the globally-supersymmetric effective theory, which turns out to be non-minimal. We also make some preliminary analysis of the differences that arise with respect to the MSSM. The problem of supersymmetry breaking and its effects on the low-energy theory, in particular on the generation of soft terms, are discussed in section 4. Armed with the complete effective theory (which now includes also supersymmetry breaking effects) we examine some phenomenological implications in later sections. Electroweak symmetry breaking is the subject of section 5, while section 6 is devoted to the analysis of the Higgs sector and section 7 to the neutralino sector. Finally, section 8 contains a summary and some conclusions. Appendix A gives the Lagrangian for a globally-supersymmetric theory with general Kähler potential and gauge kinetic function. Appendix B examines more briefly an alternative choice of Kähler potential (different from the one adopted in the main text). Appendix C contains the technical details of the calculation of the Higgs spectrum.

2 Supersymmetric warped scenarios

The starting point for supersymmetric warped constructions is the 5D SUGRA lagrangian density

$$\begin{aligned} \mathcal{L}_{\text{SUGRA},5} = & M_5^3 \left\{ \sqrt{G} \left[\frac{1}{2} R(G) - \frac{1}{4} C^{MN} C_{MN} + 6k^2 \right] \right. \\ & \left. - \frac{1}{6\sqrt{6}} \epsilon^{MNPQR} B_M C_{NP} C_{QR} + \text{fermion terms} \right\}, \end{aligned} \quad (2.1)$$

where M_5 is the 5D Planck mass; $M, N, \dots = 0, \dots, 3, 5$, are 5-dimensional space-time indices; G_{MN} is the 5-dimensional metric (with $G = \text{Det}[G_{MN}]$); $-6k^2 M_5^3$ is the bulk cosmological constant and $C_{MN} = \partial_M B_N - \partial_N B_M$ is the field strength of the graviphoton, B_M . This theory is realized on $\mathcal{M}_4 \times S_1/Z_2$ with appropriate Z_2 assignment of charges. In addition, one has to include the pieces corresponding to the two energy-densities located at the two Z_2 fixed points

$$\Delta\mathcal{L}_5 = -\delta(x^5)\sqrt{-g_1}V_1 - \delta(x^5 - \pi r_0)\sqrt{-g_2}V_2, \quad (2.2)$$

where $g_{1,2}$ are the induced 4-dimensional metrics on the boundaries and $V_{1,2}$ are constants. The initial 5D $N = 2$ SUGRA of eq. (2.1) is broken down to 4D $N = 1$ SUGRA by the boundary terms (2.2) and the Z_2 orbifold projection. Then, provided that $V_1 = -V_2 = 6M_5^3 k$, the theory admits the RS solution (1.1) with r_0 and $b_0 \equiv B_0$ arbitrary, and $B_\mu = 0$. The massless 4D fluctuations of these parameters, plus those of the 4D metric, correspond to the replacements $r_0 \rightarrow r(x)$, $b_0 \rightarrow b(x)$ and $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x)$. In fact, the corresponding metric

$$ds^2 = e^{-2kr(x)|\vartheta|} g_{\mu\nu}(x) dx^\mu dx^\nu - r^2(x) (d\vartheta)^2, \quad (2.3)$$

is shown [4] to be a solution of the 5D action (up to higher derivative terms).

The lagrangian density $\mathcal{L}_{\text{SUGRA},5} + \Delta\mathcal{L}_5$ must be supplemented with extra pieces in order to stabilize the radion field, r (and b), at some vacuum expectation values (VEVs). This is absolutely necessary for several reasons. In the absence of a potential, the radion couples to 4D gravity as a Brans-Dicke scalar with a strength incompatible with precision tests of general relativity. In addition, light weakly-coupled moduli are known to produce cosmological problems (the mass required to avoid them depends on the size of the couplings). The radion stabilization is intimately connected to the mechanism of SUSY breaking since, if SUSY is unbroken, the radion potential is flat [6].

Several models have been proposed for radion stabilization in this context. In ref. [4] this is achieved by the inclusion of two super-Yang-Mills (SYM) sectors, one in the bulk and one in the hidden brane, plus a SUSY breaking sector also located in the hidden brane. In ref. [6] stabilization is achieved by means of source superpotentials (essentially constants) located at the two branes, coupled to scalar fields living in the bulk. Actually, in ref. [6] the bulk is populated not only by the 5D gravitational sector [eq. (2.1)], but also by a number of (neutral) bulk hypermultiplets. This is reasonable if the whole picture is to be embedded in a string compactification, where moduli fields (including the universal dilaton field) abound. Finally, one has to introduce matter and gauge fields in the visible brane. In the effective 4D SUGRA theory the r and b fields combine in a complex scalar field $T = k\pi(r + ib\sqrt{2/3})$ (this can be readily checked by noting that the bulk SYM action upon integration in the fifth dimension contains the pieces $\sim r \operatorname{tr} FF + b \operatorname{tr} \tilde{F} F$ with the right coefficients [4–6]).

Following refs. [4,5], the T -dependent part of the Kähler potential in 4D SUGRA, $K(T, T^\dagger)$, can be obtained by comparing the curvature term of the action (2.1) upon integration in the compact dimension [1]

$$S_{4,\text{eff}} = \frac{1}{2} M_p^2 \int d^4x \sqrt{-g} \left[\left(1 - e^{-2\pi k r(x)}\right) R(g) + \dots \right] , \quad (2.4)$$

where $M_p^2 \equiv M_5^3/k$, with the general form of a 4D SUGRA potential

$$S_{\text{SUGRA},4} = \int d^4x d^4\theta E \phi_c^\dagger \phi_c \Phi(T, T^\dagger) = - \int d^4x \sqrt{-g} \left[\frac{1}{6} \Phi R(g) + \dots \right]. \quad (2.5)$$

Here E is the superspace determinant of the vierbein superfield, ϕ_c is the conformal compensator and $\Phi(T, T^\dagger)$ is real. The right-hand side of eq. (2.5) is written in the minimal Poincaré superconformal gauge, $\phi_c = 1 + \theta^2 F_c$. Before doing this fixing, the left hand side of eq. (2.5) is formally Weyl-invariant, so one can assume that it is written in the same conformal frame as eq. (2.4), i.e. the 4D metric is the same in both. Thus [4–6]

$$\Phi(T, T^\dagger) = 3M_p^2 \left[\exp[-(T + T^\dagger)] - 1 \right] , \quad (2.6)$$

and

$$K(T, T^\dagger) = -3M_p^2 \log \left[-\frac{1}{3M_p^2} \Phi(T, T^\dagger) \right] . \quad (2.7)$$

In addition, one has to consider the pieces coming from the two branes. Since the hidden brane corresponds to $\vartheta = 0$, the metric induced by eq. (2.3) in the hidden brane

and $\mathcal{L}_{4\text{D,hid}}$ are independent of $r(x)$. This implies T -independence of the supersymmetric theory in the hidden brane [4]. Consequently, in this conformal frame $\mathcal{L}_{4\text{D,hid}}$ must take the form [4]

$$\mathcal{L}_{4\text{D,hid}} = -\frac{1}{2} \left[\phi_c^\dagger \phi_c \Phi_{\text{hid}}(\hat{\varphi}_{\text{hid}}, \hat{\varphi}_{\text{hid}}^\dagger) \right]_D + \left[f_{\text{hid}}(\hat{\varphi}_{\text{hid}}) \text{tr}(W'^\alpha W'_\alpha) \right]_F + \left[\phi_c^3 W_{\text{hid}}(\hat{\varphi}_{\text{hid}}) \right]_F. \quad (2.8)$$

Here $\hat{\varphi}_{\text{hid}}$, W'^α are the matter and the gauge field-strength chiral superfields confined to the hidden brane; Φ_{hid} is real, and f_{hid} , W_{hid} (the gauge kinetic function and the superpotential) are holomorphic. These functions can also depend on the (even component under the Z_2 -parity of the) bulk hypermultiplets to be introduced below.

For the visible brane it is convenient to first perform a Weyl rescaling of the metric $g_{\mu\nu} \rightarrow e^{2kr(x)\pi} g_{\mu\nu}$. In this frame the induced metric in the visible brane is independent of r , thus the form of $\mathcal{L}_{4\text{D,vis}}$ becomes completely analogous to (2.8). This rescaling is equivalent to a redefinition of the conformal compensator $\phi_c \rightarrow e^T \phi_c$, prior to fixing the superconformal gauge in the usual way. Therefore, in terms of the original metric, $\mathcal{L}_{4\text{D,vis}}$ must read

$$\begin{aligned} \mathcal{L}_{4\text{D,vis}} = & -\frac{1}{2} \left[\phi_c^\dagger \phi_c e^{-(T+T^\dagger)} \Phi_{\text{vis}}(\hat{\varphi}_{\text{vis}}, \hat{\varphi}_{\text{vis}}^\dagger) \right]_D \\ & + \left[f_{\text{vis}}(\hat{\varphi}_{\text{vis}}) \text{tr}(W'^\alpha W'_\alpha) \right]_F + \left[\phi_c^3 e^{-3T} W_{\text{vis}}(\hat{\varphi}_{\text{vis}}) \right]_F. \end{aligned} \quad (2.9)$$

From eqs. (2.7–2.9) one gets the complete Kähler potential $K = -3M_p^2 \log[-\Phi/(3M_p^2)]$, where $\Phi = \Phi(T, T^\dagger) + \Phi_{\text{hid}} + e^{-(T+T^\dagger)} \Phi_{\text{vis}}$. This already shows the most distinctive features of 4D SUSY arising from warped constructions: the radion, T , is coupled to the visible fields, *both* in the superpotential and the Kähler potential, in a non-trivial (and quite model-independent) fashion. As we will see, this results in a new and characteristic phenomenology.

Eqs. (2.7–2.9) were first obtained in ref. [4], where the possible presence of additional moduli in the bulk (which seems likely, as mentioned before) was not considered. This was done in ref. [6] by representing such matter as bulk hypermultiplets. In particular, ref. [6] considered the presence of a “universal hypermultiplet” which (with a suitable choice of quaternionic sigma-model metric and couplings to the branes) reproduces in 4D the basic features of the usual dilatonic multiplet, S , of string theories: it contributes with a term $-M_p^2 \log(S + S^\dagger)$ to the Kähler potential and it is coupled to the gauge multiplets in both branes in the universal way characteristic of the dilaton,

i.e. $f_{\text{hid}} = f_{\text{vis}} = S$.

In general, we indeed expect that, in first approximation, the contribution of the hypermultiplets to the Kähler potential amounts to a separate term, say $K(S, S^\dagger)$. This is because, in the frame where eqs.(2.4, 2.5) are written, the kinetic term of S gets a factor $\Phi(T, T^\dagger)$, which simply arises from the integration in the extra dimension. Then, after a conformal redefinition to leave the curvature term in the canonical form, $\frac{1}{2}R$ (as must be done to write the SUGRA Lagrangian in the usual fashion of Cremmer et al. [9,10]) the S -kinetic term becomes T -independent. Thus, the latter must arise from an independent piece in the Kähler potential. On the other hand, it is known that in string theories the dilaton Kähler potential may acquire sizeable perturbative and non-perturbative contributions [11], which can modify its form in a substantial manner. Therefore, we prefer to leave the dilaton part of the Kähler potential, $K(S, S^\dagger)$, as an unknown function, although we will use $K(S, S^\dagger) = -M_p^2 \log(S + S^\dagger)$ in some computations with no loss of generality. With respect to the gauge kinetic functions, f , there can be important corrections to the previous universal assumption. Actually, the universal condition is not desirable in this context since it implies unification of observable gauge couplings at the TeV scale, which is phenomenologically untenable. On the other hand, non-universality of gauge couplings may arise in actual string constructions for several reasons. This happens in the context of the weakly-coupled heterotic string for different Kac-Moody levels of the various gauge groups and/or due to sizeable moduli-dependent corrections. In the context of Type I string constructions, the f -function for a given p -brane consists of a universal part (which is either the dilaton or a modulus) plus a model- and gauge-group-dependent sizeable contribution proportional to the twisted moduli. In consequence, we will not make strong assumptions about the form of $f_{\text{hid}}, f_{\text{vis}}$. Our only assumption is that f_{vis} is independent of T (which is true if the observable gauge bosons live in the visible brane). In our calculations the dilaton will only play a role in breaking supersymmetry, since this breaking will occur along the F_S -direction. However, the details about the explicit form of $K(S, S^\dagger)$ and f_{vis} turn out to be unimportant. Actually, in our set-up S may represent the ordinary dilaton or any other moduli field living in the bulk. In summary, our starting point for the effective 4D superpotential is

$$W = W_{\text{hid}} + e^{-3T} W_{\text{vis}} , \quad (2.10)$$

and for the Kähler potential²

$$K = K(S, S^\dagger) - 3M_p^2 \log \left[\frac{\Phi(T, T^\dagger) + \Phi_{\text{hid}}(\hat{\varphi}_{\text{hid}}, \hat{\varphi}_{\text{hid}}^\dagger) + e^{-(T+T^\dagger)} \Phi_{\text{vis}}(\hat{\varphi}_{\text{vis}}, \hat{\varphi}_{\text{vis}}^\dagger)}{-3M_p^2} \right] \quad (2.11)$$

where $\Phi(T, T^\dagger)$ is given by eq. (2.7). For later use we define $\Sigma_{\text{vis}}(\hat{\varphi}_{\text{vis}})$ through

$$\Phi_{\text{vis}} \equiv 3M_p^2 \left\{ \exp \left[\Sigma_{\text{vis}} / (3M_p^2) \right] - 1 \right\} , \quad (2.12)$$

so that eq. (2.11) takes the form

$$K = K(S, S^\dagger) - 3M_p^2 \log \left[1 - \exp \left\{ -(T + T^\dagger) + \frac{\Sigma_{\text{vis}}(\hat{\varphi}_{\text{vis}})}{3M_p^2} \right\} - \frac{\Phi_{\text{hid}}(\hat{\varphi}_{\text{hid}})}{3M_p^2} \right] . \quad (2.13)$$

One may wonder about the explicit form of Σ_{vis} in eq. (2.13) [or, equivalently, Φ_{vis} in eq. (2.11)]. From the point of view of 4D SUGRA it is not possible to theoretically constrain its form. On the other hand, in ref. [6] it is argued that, very likely, the actual 4D SUGRA actions from 5D are just a subset of all possible 4D SUGRA actions. In this sense, the authors of ref. [6] give an ansatz for the visible fields, consistent with the dimensional reduction from 5D, which corresponds to $\Sigma_{\text{vis}} = \sum_i |\hat{\varphi}_i|^2$. Intuitively, a justification for this is the following [6]. If we start with canonical kinetic terms [except for the $e^{-(T+T^\dagger)}$ scaling] for the $\hat{\varphi}_{\text{vis}}$ fields in the initial conformal frame [in which eqs. (2.4, 2.5) are written], then, in the limit of small k , it is easy to see that eq. (2.11) must render $K = K(S, S^\dagger) - 3M_p^2 \log \left[T + T^\dagger - |\hat{\varphi}_{\text{vis}}|^2 / (3M_p^2) \right]$. This suggests that in general the Kähler potential will depend on the T and $\hat{\varphi}_{\text{vis}}$ fields through the combination $T + T^\dagger - |\hat{\varphi}_{\text{vis}}|^2 / (3M_p^2)$, and this leads to $\Sigma_{\text{vis}} = \sum_i |\hat{\varphi}_i|^2$.

Throughout the paper we will leave Σ_{vis} as an unknown function for the general derivations, but for concrete computations we will follow ref. [6] and use the above ansatz. On the other hand, besides the $\sim |\hat{\varphi}|^2$ terms in Σ_{vis} , there may be additional “chiral” terms, like $\sim (\hat{H}_1 \cdot \hat{H}_2 + \text{h.c.})$ for the Higgses [here \cdot stands for the $SU(2)$ product: $\hat{H}_1 \cdot \hat{H}_2 \equiv i \hat{H}_1^T \boldsymbol{\sigma}_2 \hat{H}_2$]. Such terms are relevant for phenomenology and we will allow their presence. Hence, we will take

$$\Sigma_{\text{vis}} = \sum_i |\hat{\varphi}_i|^2 + \left(\lambda \hat{H}_1 \cdot \hat{H}_2 + \text{h.c.} \right) . \quad (2.14)$$

For other ansätze, most of the basic features of the non-conventional SUSY phenomenology associated to these scenarios (which will be analyzed in the following

²There is a misprint in formula (3.17) of [4] which does not have the $\exp[-(T+T^\dagger)]$ factor in front of Φ_{vis} .

sections) still hold. As stated after eq.(2.9), this is a consequence of the non-trivial and quite model-independent way in which the radion is coupled to the observable fields in K and W . This holds in particular if one takes the somewhat simpler ansatz $\Phi_{\text{vis}} = \sum_i |\hat{\varphi}_i|^2$. This case is briefly discussed in Appendix B.

With regard to the dependence of W_{vis} on the observable fields, we will consider a MSSM-like superpotential, with the ordinary Yukawa terms plus a μ mass term for the Higgses (possible non-renormalizable terms will be discussed later on):

$$W_{\text{vis}}(\hat{\varphi}_{\text{vis}}) = \hat{h}_U^{ij} \hat{Q}_{Li} \cdot \hat{H}_2 \hat{U}_{Rj} + \hat{h}_D^{ij} \hat{H}_1 \cdot \hat{Q}_{Li} \hat{D}_{Rj} + \hat{\mu}_0 \hat{H}_1 \cdot \hat{H}_2 . \quad (2.15)$$

where $i, j = 1, 2, 3$ are family indices. The value of $\hat{\mu}_0$ is naturally of the order of the fundamental scale of the theory, i.e. $\hat{\mu}_0 = \mathcal{O}(M_p)$.

3 Effective supersymmetric theory

In this section we derive the effective globally-supersymmetric theory at low-energies that arises from the 4D superpotential and Kähler potential displayed in section 2, eqs. (2.11–2.15). The contributions from SUSY breaking will be studied later, in sect. 4.

In order to extract the effective theory, we have first to take into account that the radion field, T , takes an expectation value such that

$$e^{-\langle T+T^\dagger \rangle} \equiv \frac{\Lambda^2}{3M_p^2} , \quad (3.1)$$

where $e^{-\langle T+T^\dagger \rangle}$ is the usual warp suppression factor and $\Lambda = \mathcal{O}(\text{TeV})$ is essentially the effective fundamental mass-scale in the visible brane. This means, in particular, that T and the visible fields $\hat{\varphi}_i$ (we will remove the subscript “vis” if there is no risk of confusion) in eqs. (2.11–2.15) are far from being correctly normalized. We define t , φ_i as fields with canonical kinetic terms (before electroweak breaking) by

$$T = \langle T \rangle + t/\Lambda , \quad \varphi_i = \frac{\Lambda}{\sqrt{3}M_p} \hat{\varphi}_i . \quad (3.2)$$

Then, the effective Kähler potential at low energy is obtained from eq. (2.13) by taking the limit $M_p \rightarrow \infty$ (with Λ fixed). Focusing just on the observable sector, this gives

$$\boxed{K_{\text{eff}} = \Lambda^2 \exp \left\{ -\frac{t+t^*}{\Lambda} + \frac{1}{\Lambda^2} \Sigma_{\text{eff}}(\varphi_i) \right\}} \quad (3.3)$$

where $\Sigma_{\text{eff}} \equiv (\Lambda^2/3M_p^2)\Sigma_{\text{vis}}$. Since it is quadratic in the fields, Σ_{eff} has the same form as Σ_{vis} , eq. (2.14), in terms of the canonically normalized fields:

$$\Sigma_{\text{eff}} = \sum_i |\varphi_i|^2 + (\lambda H_1 \cdot H_2 + \text{h.c.}) \quad (3.4)$$

The low energy effective superpotential in the observable sector is given by [12] $W_{\text{eff}} = e^{\langle K/2M_p^2 \rangle} e^{-3T} W_{\text{vis}}$. So, using eqs. (2.15, 3.1, 3.2), we get

$$W_{\text{eff}} = e^{-3t/\Lambda} \left[h_U^{ij} Q_{Li} \cdot H_2 U_{Rj} + h_D^{ij} H_1 \cdot Q_{Li} D_{Rj} + \mu_0 H_1 \cdot H_2 \right] \quad (3.5)$$

with

$$\begin{aligned} h_{U,D}^{ij} &= e^{\langle K/2M_p^2 \rangle} e^{-3i\langle \text{Im}T \rangle} \hat{h}_{U,D}^{ij} = \mathcal{O}(1) \hat{h}_{U,D}^{ij} , \\ \mu_0 &= e^{\langle K/2M_p^2 \rangle} e^{-3i\langle \text{Im}T \rangle} \left(\frac{\Lambda}{\sqrt{3}M_p} \right) \hat{\mu}_0 = \mathcal{O}(\Lambda) , \end{aligned} \quad (3.6)$$

where $e^{\langle K/2M_p^2 \rangle} \simeq \mathcal{O}(1)$. Equation (3.6) illustrates the fact that, once the fields are properly normalized, all the mass scales in the observable sector (in this case the μ term) get a suppression factor $\mathcal{O}(\Lambda/M_p)$. This in particular means that there is no μ -problem in this context, since μ_0 has naturally the right order of magnitude.

W_{eff} and K_{eff} in (3.5) and (3.3) describe a globally-supersymmetric theory valid at energies below the scale Λ , which plays the role of UV cut-off³. Physically, Λ corresponds roughly to the mass scale of the lightest Kaluza-Klein excitations⁴. The supersymmetric Lagrangian (before SUSY breaking) that follows from (3.5) and (3.3) can be obtained in terms of component fields using the general formulae for non-minimal Kähler potentials given in Appendix A.

If the scale Λ were much higher than μ_0 , this scenario would reproduce the MSSM: an expansion in powers of μ_0/Λ and φ_i/Λ gives

$$W_{\text{eff}} \simeq h_U^{ij} Q_{Li} \cdot H_2 U_{Rj} + h_D^{ij} H_1 \cdot Q_{Li} D_{Rj} + \mu_0 H_1 \cdot H_2 + \dots \equiv W_{\text{MSSM}} + \dots , \quad (3.7)$$

and

$$K_{\text{eff}} \simeq \Lambda^2 - \Lambda(t + t^*) + \frac{1}{2}(t + t^*)^2 + |H_1|^2 + |H_2|^2 + (\lambda H_1 \cdot H_2 + \text{h.c.}) + \dots \quad (3.8)$$

³Non-minimal Kähler potentials that result from decoupling heavy fields are discussed in [13].

⁴More precisely, the mass of the lightest K-K excitations is [1,4] $m_{KK} \simeq \frac{\Lambda k}{\sqrt{3}M_p} = \frac{\Lambda}{\sqrt{3}} \left(\frac{k}{M_5} \right)^{3/2}$.

with terms suppressed by inverse powers of Λ omitted. Throwing away terms that give a zero contribution to the D -term of K_{eff} (and so, do not contribute to the Lagrangian), the Kähler potential in (3.8) is equivalent to the simpler minimal one

$$K_{\text{eff}} \simeq |t|^2 + |H_1|^2 + |H_2|^2 + \dots \equiv K_{\text{min}} + \dots \quad (3.9)$$

In this limit, therefore, the Lagrangian reduces to that of the MSSM (the radion field, t , simply decouples from the Standard fields).

We are, of course, interested in a different regime with $\Lambda \sim 4\text{--}10$ TeV. The lower limit⁵ is roughly what is required to satisfy experimental limits on exotic KK excitations of gravitational fields [14]. The upper limit is set by naturalness criteria, as we wish to avoid reintroducing a hierarchy problem. Therefore, Λ should not be much larger than μ_0 (or than the gravitino mass, $m_{3/2}$, as is discussed in the next section). In this case, K_{eff} and W_{eff} , as given in eqs. (3.3, 3.5), do not reduce to the minimal ones, eqs. (3.7, 3.9), so we expect important deviations from ordinary supersymmetric versions of the SM, like the MSSM or the NMSSM [15]. Such deviations are due to

- i) The presence of new light degrees of freedom, associated to the radion superfield, t .
- ii) Mixing of t with the Higgs fields through kinetic and mass terms after electroweak symmetry breaking. A similar mixing occurs for the fermionic partners.
- iii) Couplings that deviate from the MSSM ones by corrections suppressed only by powers of μ_0/Λ (or $m_{3/2}/\Lambda$ after SUSY breaking). These include, in particular, new quartic couplings for the Higgses.
- iv) Higher order operators (*e.g.* with two derivatives), suppressed only by inverse powers of Λ . Incidentally, the ultraviolet cut-off $\sim \Lambda$ is much smaller than in more conventional SUSY scenarios.

The main effects arise in the Higgs effective potential, the kinetic terms and Yukawa couplings. We consider each in turn:

⁵The precise value for this lower limit depends on the value of k/M_p , see [14]. The quoted value corresponds to $k/M_p \sim 0.5$.

3.1 Supersymmetric Higgs potential

The scalar potential of a SUSY theory with a general Kähler potential, K_{eff} , and a general gauge kinetic-function, f_A , can be obtained from the general Lagrangian presented in Appendix A, after using $f_{AB}(\phi) = \delta_{AB}f_A(\phi)$ and eliminating the F and D -terms. It is given by (sum over repeated indices is always implied)

$$V_{\text{SUSY}} = \mathcal{G}_{IJ}^{-1} \left(\frac{\partial W_{\text{eff}}}{\partial \phi_I} \right)^* \frac{\partial W_{\text{eff}}}{\partial \phi_J} + \frac{1}{4} \left[f_A^{-1}(\phi) \left(\frac{\partial K_{\text{eff}}}{\partial \phi_I} [\mathbf{t}_{\mathbf{A}}]_{IJ} \phi_J \right)^2 + \text{h.c.} \right], \quad (3.10)$$

where ϕ_I runs over all the chiral fields of the theory $\{\varphi_i, t, S\}$; $\mathbf{t}_{\mathbf{A}}$ are the matrix generators of the gauge algebra in the ϕ_J representation (S and t , being singlets, do not contribute to the D -terms), and \mathcal{G}_{IJ}^{-1} is the inverse matrix of the Kähler metric

$$\mathcal{G}_{IJ} \equiv \frac{\partial^2 K_{\text{eff}}}{\partial \phi_I \partial \phi_J^*}. \quad (3.11)$$

With our assumptions on the S -dependence of K (see previous section), \mathcal{G}_{IJ} decomposes in block form, with $\mathcal{G}_{St} = \mathcal{G}_{S\varphi_i} = 0$. [In fact, S does not contribute as a dynamical field to the potential (3.10), which has been obtained in the $M_p \rightarrow \infty$ limit.] The non-trivial part of the inverse matrix \mathcal{G}_{IJ}^{-1} corresponds to the $\{\varphi_i, t\}$ block. In terms of Σ_{eff} , this inverse (sub)matrix is

$$\mathcal{G}_{IJ}^{-1} = \frac{\Lambda^2}{K_{\text{eff}}} \begin{bmatrix} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j^*} \right]^{-1} & \frac{1}{\Lambda} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_k^*} \right]^{-1} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_k} \\ \frac{1}{\Lambda} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_k^*} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_k \partial \varphi_j^*} \right]^{-1} & 1 + \frac{1}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_k^*} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_k \partial \varphi_l^*} \right]^{-1} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_l} \end{bmatrix}. \quad (3.12)$$

where $[\partial^2 \Sigma_{\text{eff}} / \partial \varphi_i \partial \varphi_j^*]^{-1}$ represents the inverse matrix of $\partial^2 \Sigma_{\text{eff}} / \partial \varphi_i \partial \varphi_j^*$. For Σ_{eff} as defined in (3.4) one simply has $[\partial^2 \Sigma_{\text{eff}} / \partial \varphi_i \partial \varphi_j^*]^{-1} = \delta_{ij}$ but, for later use, we find it convenient to leave the general expressions in terms of derivatives of Σ_{eff} . With the help of (3.12), the potential (3.10) can be rewritten as

$$V_{\text{SUSY}} = \frac{\Lambda^2}{K_{\text{eff}}} \left\{ \left(\frac{\partial W_{\text{eff}}}{\partial \phi_i} - \frac{3W_{\text{eff}}}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} \right)^* \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j^*} \right]^{-1} \left(\frac{\partial W_{\text{eff}}}{\partial \phi_j} - \frac{3W_{\text{eff}}}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_j} \right) + \frac{9}{\Lambda^2} |W_{\text{eff}}|^2 \right\} + \frac{K_{\text{eff}}^2}{4\Lambda^4} \left\{ f_A^{-1}(\phi) \left[\frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} (\mathbf{t}_{\mathbf{A}} \varphi)_i \right]^2 + \text{h.c.} \right\}. \quad (3.13)$$

Concerning the D -term part of the potential, note that, if Σ_{eff} contains terms of the form $a(\varphi) + a(\varphi^*)$ [like the λ -dependent part of (3.4)], gauge invariance implies

$$\frac{\partial a(\varphi)}{\partial \varphi_i} (\mathbf{t}_{\mathbf{A}} \varphi)_i = 0, \quad (3.14)$$

so that $a(\varphi)$ contributes to the D -terms in V only through the pre-factor K_{eff} , giving therefore only non-renormalizable (Λ -suppressed) contributions.

The supersymmetric part of the potential for the Higgs fields, relevant to the study of electroweak symmetry breaking, is given [after plugging (3.3,3.5) and $f_A = g_A^{-2}$ in (3.13)] by

$$\begin{aligned}
V_{\text{SUSY}}(H_i, t) = & \exp \left\{ -2 \frac{t+t^*}{\Lambda} - \frac{1}{\Lambda^2} \left[|H_1|^2 + |H_2|^2 + (\lambda H_1 \cdot H_2 + \text{h.c.}) \right] \right\} \\
& \times \left\{ -3 \frac{|\mu_0|^2}{\Lambda^2} |H_1 \cdot H_2|^2 \left[1 - \frac{6}{\Lambda^2} (\lambda H_1 \cdot H_2 + \text{h.c.}) \right] \right. \\
& + |\mu_0|^2 \left[\left| 1 - \frac{3}{\Lambda^2} \lambda H_1 \cdot H_2 \right|^2 + \frac{9}{\Lambda^4} |H_1 \cdot H_2|^2 \right] (|H_1|^2 + |H_2|^2) \Big\} \\
& + \exp \left\{ -2 \frac{t+t^*}{\Lambda} + \frac{2}{\Lambda^2} \left[|H_1|^2 + |H_2|^2 + (\lambda H_1 \cdot H_2 + \text{h.c.}) \right] \right\} \\
& \times \left\{ \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g^2 (|H_1|^2 |H_2|^2 - |H_1 \cdot H_2|^2)^2 \right\} ,
\end{aligned} \tag{3.15}$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants and

$$H_1 = \begin{bmatrix} H_1^0 \\ H_1^- \end{bmatrix} , \quad H_2 = \begin{bmatrix} H_2^+ \\ H_2^0 \end{bmatrix} , \tag{3.16}$$

are the usual two Higgs doublets.

One noteworthy feature of this potential is that $\text{Re}[t]$ exhibits a runaway behaviour (if $H_{1,2}$ take non-zero VEVs). At this level, that is inconsistent with our replacement $T = \langle T \rangle + t/\Lambda$, which assumes T is stabilized at $\langle T \rangle$ as given by (3.1). In fact, that stabilization occurs only after SUSY is broken [6], as we discuss in the next section. For the time being, we simply ignore the problem and examine other peculiarities of V_{SUSY} , limiting our analysis to its H_i -dependent part and ignoring the radion field. As expected, for $\Lambda \rightarrow \infty$ the radion field gets decoupled and eq. (3.15) reduces to the ordinary MSSM potential. An expansion of (3.15) in powers of H_i/Λ , keeping only renormalizable terms, gives:

$$\begin{aligned}
V(H_i) = & |\mu_0|^2 (|H_1|^2 + |H_2|^2) + \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 \\
& + \frac{1}{2} g^2 (|H_1|^2 |H_2|^2 - |H_1 \cdot H_2|^2) - \frac{|\mu_0|^2}{\Lambda^2} \left[(|H_1|^2 + |H_2|^2)^2 \right. \\
& + 4 (|H_1|^2 + |H_2|^2) (\lambda H_1 \cdot H_2 + \text{h.c.}) + 3 |H_1 \cdot H_2|^2 \Big] + \dots
\end{aligned} \tag{3.17}$$

Besides the usual MSSM quartic couplings, related to gauge couplings, this potential contains additional contributions which are purely supersymmetric and arise from the non-minimal nature of the Kähler potential and the coupling $tH_1 \cdot H_2$ in W_{eff} .

At first sight one might worry about the stability of the vacuum with a potential like (3.17). In fact, along the D -flat direction $\varphi \equiv \text{Re}(H_1^0 + H_2^0)$, the potential has a quartic coupling that can be negative for some choices of λ :

$$V(\varphi) = \frac{1}{2}|\mu_0|^2\varphi^2 - \frac{1}{16}\frac{|\mu_0|^2}{\Lambda^2}(7 + 8\lambda + 8\lambda^*)\varphi^4 + \dots \quad (3.18)$$

However, this behaviour is an artifact of the φ/Λ -expansion. The full potential (3.15) along the D -flat direction,

$$V(\varphi) = \frac{1}{8}|\mu_0|^2\varphi^2 \left[\frac{9}{2}\frac{\varphi^2}{\Lambda^2} + \left| 2 - 3(1 + \lambda)\frac{\varphi^2}{2\Lambda^2} \right|^2 \right] \exp \left\{ - \left[1 + \frac{1}{2}(\lambda + \lambda^*) \right] \frac{\varphi^2}{2\Lambda^2} \right\} , \quad (3.19)$$

is not only bounded from below but also positive definite for any value of λ , as it should. In fact this holds for any other field direction and means, in particular, that the electroweak symmetry is unbroken until supersymmetry-breaking effects are considered, just like in the MSSM.

As we will discuss in section 4, the presence of new contributions to Higgs quartic couplings has a dramatic impact on the value of the lightest Higgs boson mass. It would be premature at this stage to extract implications on this respect from (3.15) because one expects that supersymmetry breaking physics also gives rise to new Higgs quartic couplings of order $m_{3/2}^2/\Lambda^2$ which can compete with the supersymmetric ones just presented. In fact, the presence of such adimensional SUSY-breaking terms and its effect on the Higgs mass have been discussed in ref. [16] for a variety of models with low-energy supersymmetry breaking (see also [17]). Let us remark again that a novel feature of our scenario is that it gives new contributions to Higgs quartic couplings which, unlike those studied in [16], are truly supersymmetric and arise from the non-minimal Kähler potential.

Before closing this subsection, we mention another possible source of non-standard supersymmetric contributions to Higgs quartic couplings, although we do not consider it for the discussion of the phenomenological implications. Suppose that the original superpotential (2.15) contains non-renormalizable terms like

$$\delta W_{\text{vis}} = \frac{\hat{c}}{\sqrt{3}M_p}(\hat{H}_1 \cdot \hat{H}_2)^2 . \quad (3.20)$$

After changing to canonically normalized fields and shifting the radion field T , we get a contribution to the effective superpotential of the form

$$\delta W_{\text{eff}} = \frac{c}{\Lambda} (H_1 \cdot H_2)^2 e^{-3t/\Lambda} , \quad (3.21)$$

with $c \equiv \hat{c} e^{\langle K/2M_p^2 \rangle} e^{-3i\langle \text{Im} T \rangle}$. It is straightforward to show that (3.21) contributes to the effective potential (3.17) the additional renormalizable term

$$\delta V_{\text{SUSY}} = 2 \left(c \frac{\mu_0^*}{\Lambda} H_1 \cdot H_2 + \text{h.c.} \right) (|H_1|^2 + |H_2|^2) , \quad (3.22)$$

which is of the same order as that of the quartic terms considered in (3.17), unless c is very small.

3.2 Kinetic terms, contributions to the ρ -parameter

A non-minimal Kähler potential like (3.3) leads, after electroweak symmetry breaking, to non-canonical kinetic terms for scalar fields (see Appendix A):

$$\mathcal{L}_{\text{kin}} = \langle \mathcal{G}_{IJ} \rangle D_\mu \phi_I D^\mu \phi_J^* , \quad (3.23)$$

where the brackets indicate that all fields are replaced by their VEVs. $\langle \mathcal{G}_{IJ} \rangle$ is in general a non-diagonal matrix, in particular for the entries involving the Higgses and the radion, due to $\mathcal{O}(v/\Lambda)$ terms:

$$\langle \mathcal{G}_{IJ} \rangle = \delta_{IJ} + \mathcal{O} \left(\frac{v}{\Lambda} \right) , \quad (3.24)$$

where v denote the Higgs VEVs. Getting the kinetic terms back to canonical form requires a re-scaling and redefinition of fields, which affects other couplings in the Lagrangian. The implications of such effects on the composition and masses of the Higgs fields are discussed in detail in section 6 and Appendix C. There are also implications for gauge boson masses. Explicitly, (3.23) gives, for the Higgs kinetic terms:

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \frac{\langle K_{\text{eff}} \rangle}{\Lambda^2} \left[|D_\mu H_1|^2 + |D_\mu H_2|^2 \right] \\ &\quad + \frac{\langle K_{\text{eff}} \rangle}{\Lambda^4} \left| (\overline{H}_1 + \lambda H_2) \cdot D_\mu H_1 + (\overline{H}_2 - \lambda H_1) \cdot D_\mu H_2 \right|^2 , \end{aligned} \quad (3.25)$$

where K_{eff} is given by eq. (3.3) and \overline{H}_i is the $SU(2)$ -conjugate of H_i ($\overline{H}_i \equiv -i\sigma_2 H_i^*$; with $|H_i|^2 \equiv H_i \cdot \overline{H}_i$). From the expression above, it is straightforward to obtain the gauge boson masses:

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2 \frac{\langle K_{\text{eff}} \rangle}{\Lambda^2} \left[1 + \frac{v^2}{2\Lambda^2} \cos^2 2\beta \right] , \quad (3.26)$$

and

$$M_W^2 = \frac{1}{4} g^2 v^2 \frac{\langle K_{\text{eff}} \rangle}{\Lambda^2} , \quad (3.27)$$

with $v^2 = 2(|H_1|^2 + |H_2|^2)$, $\tan \beta \equiv \langle H_2^0 \rangle / \langle H_1^0 \rangle$ and

$$\frac{\langle K_{\text{eff}} \rangle}{\Lambda^2} = 1 + 2(1 + \lambda \sin 2\beta) \frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right) . \quad (3.28)$$

From the masses (3.26) and (3.27) we first see that there is a deviation of v from its minimal value ~ 246 GeV. That deviation is a small $\mathcal{O}(v^2/\Lambda^2)$ effect that we neglect in the following. Then we see that there is also a tree-level deviation from $\rho = 1$, with $\Delta\rho \simeq -\frac{1}{2}(v^2/\Lambda^2) \cos^2 2\beta$, which can be traced back to the terms in the second line of (3.25). Consequently, the comparison to the present measurements of the ρ parameter translates into a bound on Λ . The precise value of the latter depends on the value of the Higgs mass, M_{h^0} , because the larger M_{h^0} , the lower the SM prediction for ρ . Since the above tree-level contribution is negative, the corresponding bound on Λ becomes even stronger for larger M_{h^0} . In order to give a more quantitative estimate of the bound, notice that the 2σ experimental window (with M_h^0 unconstrained), $\Delta\rho = 0.9998^{+0.0034}_{-0.0012}$ [18], leaves little room for non SM contributions, and we get the bound

$$\Lambda \gtrsim (5.5 \text{ TeV}) |\cos 2\beta| . \quad (3.29)$$

As we will show in section 4, a correct breaking of the electroweak symmetry in this context normally demands $\tan \beta \simeq 1$ and, therefore, this source of non-zero $\Delta\rho$ and the previous upper bound disappear.

The tree-level behaviour of ρ for $\tan \beta = 1$ can be understood by symmetry arguments. Let us define the bi-doublets

$$\mathbf{H} = \begin{bmatrix} H_1^0 & H_2^+ \\ H_1^- & H_2^0 \end{bmatrix} , \quad \overline{\mathbf{H}} = \begin{bmatrix} H_2^{0*} & -H_1^+ \\ -H_2^- & H_1^{0*} \end{bmatrix} , \quad (3.30)$$

built out of the two Higgs doublets H_1 and H_2 (and their conjugates $\overline{H}_{1,2}$). In terms of \mathbf{H} and $\overline{\mathbf{H}}$, the kinetic Lagrangian (3.25) can be written as

$$\mathcal{L}_{\text{kin}} = \frac{\langle K_{\text{eff}} \rangle}{\Lambda^2} \text{Tr} \left[(D_\mu \mathbf{H})^\dagger D^\mu \mathbf{H} \right] + \frac{\langle K_{\text{eff}} \rangle}{\Lambda^4} \left| \text{Tr} \left[(\mathbf{H} - \lambda \overline{\mathbf{H}})^\dagger D_\mu \mathbf{H} \right] \right|^2 . \quad (3.31)$$

This expression makes manifest the approximate invariance of \mathcal{L}_{kin} under the $SU(2)_L \times SU(2)_R$ (global) group of transformations:

$$\mathbf{H} \rightarrow \mathbf{U}_L \mathbf{H} \mathbf{U}_R^\dagger , \quad (3.32)$$

$$\mathbf{W}_\mu \equiv \boldsymbol{\sigma}^a W_\mu^a \rightarrow \mathbf{U}_L \mathbf{W}_\mu \mathbf{U}_L^\dagger , \quad (3.33)$$

where $\mathbf{U}_{\mathbf{L},\mathbf{R}}$ are generic 2×2 $SU(2)$ matrices (and $\boldsymbol{\sigma}^a$ are the Pauli matrices). To see this, notice that (3.32) implies that $\bar{\mathbf{H}}$ transforms as \mathbf{H} :

$$\bar{\mathbf{H}} \rightarrow \mathbf{U}_{\mathbf{L}} \bar{\mathbf{H}} \mathbf{U}_{\mathbf{R}}^\dagger, \quad (3.34)$$

while (3.33) implies that, for $g' = 0$,

$$D_\mu \mathbf{H} \rightarrow \mathbf{U}_{\mathbf{L}} (D_\mu \mathbf{H}) \mathbf{U}_{\mathbf{R}}^\dagger. \quad (3.35)$$

This approximate $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken by the VEVs of H_1^0 and H_2^0 down to a subgroup G . In the case $\langle H_1^0 \rangle = \langle H_2^0 \rangle$, *i.e.* for $\tan \beta = 1$, the unbroken symmetry G is the diagonal $SU(2)_{L+R}$ (with $\mathbf{U}_{\mathbf{L}} = \mathbf{U}_{\mathbf{R}}$), the so-called custodial symmetry [19]. [The generator of the unbroken electromagnetic $U(1)_Q$ corresponds to σ_3 of this $SU(2)$.] The electroweak-symmetry-breaking Goldstone bosons, $\{G^+, G^0, G^-\}$, transform as a triplet under this custodial symmetry, ensuring $\rho = 1$. This symmetry plays an important role for the Higgs spectrum as we discuss in detail in section 6. If $\langle H_1^0 \rangle \neq \langle H_2^0 \rangle$, the unbroken subgroup G is simply $U(1)_Q$ and $\rho \neq 1$.

3.3 Yukawa couplings, fermion masses

At the renormalizable level, the only difference in fermionic couplings in our Lagrangian with respect to the MSSM one is the presence of Yukawa couplings that mix the standard Higgsinos with the fermionic component of the radion superfield (the “radino”). From the general Lagrangian written in Appendix A, after eliminating the auxiliary fields, we find, for the two-fermion couplings (with no derivatives):

$$\delta \mathcal{L}_{\text{SUSY}} = \frac{1}{2} \left\{ -\frac{\partial^2 W_{\text{eff}}}{\partial \phi_I \partial \phi_J} + \frac{\partial^3 K_{\text{eff}}}{\partial \phi_I \partial \phi_J \partial \phi_L^*} \mathcal{G}_{LK}^{-1} \frac{\partial W_{\text{eff}}}{\partial \phi_K} \right\} (\chi_I \cdot \chi_J) + \text{h.c.}, \quad (3.36)$$

where the Weyl spinors $\{\chi_I\} = \{\chi_i, \chi_t\}$ are the fermionic partners of the scalars $\{\phi_I\} = \{\varphi_i, t\}$, and $(\chi_I \cdot \chi_J) = i \chi_I^T \boldsymbol{\sigma}_2 \chi_J$. Using (3.12) we can rewrite the previous expression in the form

$$\begin{aligned} \delta \mathcal{L}_{\text{SUSY}} &= \frac{1}{2} \left\{ -\frac{\partial^2 W_{\text{eff}}}{\partial \varphi_i \partial \varphi_j} + \frac{2}{\Lambda} \frac{\partial W_{\text{eff}}}{\partial \varphi_i} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_j} + \frac{3W_{\text{eff}}}{\Lambda^2} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j} - \frac{1}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_j} \right] \right. \\ &\quad + \frac{\partial^3 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j \partial \varphi_l^*} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_l \partial \varphi_k^*} \right]^{-1} \left(\frac{\partial W_{\text{eff}}}{\partial \varphi_k} - \frac{3W_{\text{eff}}}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_k} \right) \left. \right\} (\chi_i \cdot \chi_j) \\ &\quad + \frac{2}{\Lambda} \frac{\partial W_{\text{eff}}}{\partial \varphi_i} (\chi_i \cdot \chi_t) - \frac{3W_{\text{eff}}}{\Lambda^2} (\chi_t \cdot \chi_t) + \text{h.c.} \end{aligned} \quad (3.37)$$

Explicitly, this leads to the following Yukawa couplings (and fermionic mass terms):

$$\begin{aligned} \delta\mathcal{L}_{\text{SUSY}} &= \mu_0 e^{-3t/\Lambda} \left[\chi_{H_1^-} \cdot \chi_{H_2^+} - \chi_{H_1^0} \cdot \chi_{H_2^0} \right. \\ &\quad \left. + \frac{2}{\Lambda} \left(H_2^0 \chi_{H_1^0} + H_1^0 \chi_{H_2^0} - H_2^+ \chi_{H_1^-} - H_1^- \chi_{H_2^+} \right) \cdot \chi_t \right] + \text{h.c.} \end{aligned} \quad (3.38)$$

in addition to the MSSM $h_{U,D}$ -Yukawa couplings which we do not write. After electroweak symmetry breaking, (3.38) implies that the 5×5 neutralino mass matrix contains mixing terms between $\chi_{H_i^0}$ and χ_t . To write the complete matrix we need to introduce also the effects from SUSY breaking, which are most important for gauginos and χ_t . These effects are discussed in the next section.

4 Supersymmetry breaking

4.1 The mechanism of SUSY breaking

As mentioned in section 2, supersymmetry breaking plays an essential role in the stabilization of the radion field. In addition, it is mandatory for a correct low-energy phenomenology. In the framework of ref. [6], the breaking of SUSY is triggered by a brane superpotential of the form

$$W_{sbr} = W_h + e^{-3T} W_v, \quad (4.1)$$

where W_h and W_v are constant brane sources (at $x^5 = 0$ and $x^5 = \pi r_0$, respectively). As was shown there, the radion field gets stabilized in this process, although with a non-vanishing (negative) cosmological constant in the 4-D effective theory. On the other hand, the dilaton field presents a runaway behaviour. These are shortcomings for a viable phenomenology.

Here we will discuss the above-mentioned issues: SUSY breaking, stabilization of T and S , and the value of the cosmological constant. In the next subsection we will extract and discuss the form of the soft terms. We will not make any initial assumptions about the possible dependence of W_{sbr} on S and the form of $K(S, S^\dagger)$. Actually, W_{sbr} will have a non-trivial S -dependence if it is generated by gaugino condensation (recall that the ordinary dilaton is intimately related to the gauge coupling constant). On the other hand, in the context of string theory we expect large perturbative and non-perturbative corrections to the Kähler potential [11].

To study gravity-mediated SUSY-breaking effects we have to consider the whole SUGRA potential [rather than (3.10)]. First, to discuss how W_{sbr} breaks SUSY we consider the SUGRA effective potential for T and S , neglecting matter fields (the effects of electroweak symmetry breaking represent small corrections in this analysis). This potential reads [9]

$$V_{\text{SUGRA}}(T, S) = \exp(K/M_p^2) \left[\sum_{I,J=T,S} L_I^* \mathcal{G}_{IJ}^{-1} L_J - 3 \frac{|W|^2}{M_p^2} \right], \quad (4.2)$$

with K the Kähler potential, \mathcal{G}_{IJ} its associated metric [as defined by (3.11)], W the superpotential (4.1), and

$$L_J \equiv \frac{\partial W}{\partial \phi_J} + \frac{W}{M_p^2} \frac{\partial K}{\partial \phi_J}. \quad (4.3)$$

Alternatively,

$$V_{\text{SUGRA}}(T, S) = \sum_{I,J=T,S} F_I \mathcal{G}_{IJ} F_J^* - 3 \frac{|W|^2}{M_p^2} \exp(K/M_p^2), \quad (4.4)$$

with

$$F_I \equiv \exp(K/2M_p^2) L_K^* \mathcal{G}_{KI}^{-1}. \quad (4.5)$$

Provided K can be decomposed as $K = K(S, S^\dagger) + K(T, T^\dagger)$ (which is the most plausible case, as argued in sect. 2) and W_{sbr} has a factorizable dependence on S , V presents a stationary point in T at

$$F_T \equiv \exp[K/(2M_p^2)] \mathcal{G}_{TT}^{-1} L_T^* = 0. \quad (4.6)$$

This stationary point will often correspond to a minimum in T . This is guaranteed, in particular, if V is vanishing at that point. Using the explicit form for $K(T, T^\dagger)$ given in Eq. (2.13), then Eq. (4.6) is equivalent to the minimization condition $\partial V / \partial T = 0$, which gives [6]:

$$W_v \exp[-3\langle T \rangle] + W_h \exp[-\langle T + T^\dagger \rangle] = 0. \quad (4.7)$$

Now, the parameters in the brane superpotential (4.1) can be arranged so as to obtain the necessary hierarchy (3.1), that is, $\exp[-\langle T + T^\dagger \rangle] = \Lambda^2/(3M_p^2)$, which follows if $\sqrt{3}|W_h|M_p = |W_v|\Lambda$. In addition, one should have $W_v \sim M_p^3$, so that $\langle W_{sbr} \rangle$ has the right order of magnitude for a sensible gravitino mass, as we shall see shortly. The VEV for the imaginary part of T is determined by the relative phase between W_h and W_v .

The stabilization of S is not so straightforward: as mentioned above, for W_h, W_v constant and $K(S, S^\dagger) = -M_p^2 \log(S + S^\dagger)$, S exhibits a runaway behaviour. If, following ref. [6], one fixes the dilaton by hand at a given value, then the potential has the minimum at negative V , which jeopardizes a sensible phenomenology. So, in order to proceed we have to modify the scenario so that S may be stabilized at a finite value by some unknown mechanism while the cosmological constant vanishes. This could be accomplished in several ways. For instance, if $K = K(S, S^\dagger) + K(T, T^\dagger)$ with $K(S, S^\dagger) = -3M_p^2 \ln(S + S^\dagger)$, as in no-scale models [21], this choice guarantees a zero cosmological constant for all values of S [which means in particular that the condition $F_T = 0$, eq. (4.6), corresponds to a true minimum]. This form of $K(S, S^\dagger)$ is perfectly plausible. Recall that S may represent the ordinary dilaton or any other moduli field living in the bulk. In particular, in the context of string theory, it may represent one of the moduli which parametrize the size and shape of the six extra compactified dimensions. In the overall modulus approximation (i.e. all extra radii equal) the Kähler potential is exactly as the one required [22]. There is however a problem related to this scenario. Namely, the potential has flat directions along the real and imaginary parts of S , with the corresponding massless particles in the spectrum. Eventually, non-perturbative corrections could lift these flat directions, fix $\langle S \rangle$ and give a (light) mass to these fields. As will be shown shortly, the couplings of the S field to matter fields are suppressed by inverse powers of M_p . The presence of such light weakly-interacting scalar fields could be very dangerous for a viable cosmology [23], although, definitely, the subject deserves further study.

Here we take a different route and assume that (4.1) has also an S -dependence (at this point we deviate from the assumptions of [6]; for alternative options see [24]) while keeping $K(S, S^\dagger) = -M_p^2 \ln(S + S^\dagger)$.⁶ Since $F_T = 0$, the condition for $V(\langle T \rangle, \langle S \rangle) = 0$ reads

$$\left\langle \left| (S + S^\dagger) \frac{\partial W}{\partial S} - W \right|^2 \right\rangle = 3 \langle |W|^2 \rangle . \quad (4.8)$$

If the dependence of W_{sbr} on S is factorizable⁷, i.e. $W_{sbr} \propto f(S)$, then eq. (4.8) is

⁶Although we do not continue analyzing here the no-scale possibility, we will see at the end of the section that it leads to results very similar to those arising from the present assumptions.

⁷The hypothesis of factorizability is by no means necessary, but it is convenient since, as discussed above, it guarantees $F_T = 0$ at the minimum, preserving the cancellation of the cosmological constant. On the other hand, notice that in this case the S -dependence of W may be moved into K by a Kähler transformation.

equivalent to

$$\left\langle (S + S^\dagger) \frac{1}{f} \frac{\partial f}{\partial S} \right\rangle \equiv \varsigma = 1 + \sqrt{3}e^{i\alpha} , \quad (4.9)$$

with α an arbitrary phase. We will assume this in the following. Obviously we are not solving the cosmological constant problem but simply making the necessary adjustments to ensure such cancellation.

In summary, we are therefore led to a scenario with supersymmetry breaking driven by the dilaton, with

$$F_S \equiv \exp[K/(2M_p^2)] \mathcal{G}_{SS}^{-1} L_S^* \neq 0 , \quad (4.10)$$

and cosmological constant tuned to zero. The resulting gravitino mass is

$$m_{3/2} = \exp[\langle K \rangle / (2M_p^2)] \frac{|\langle W \rangle|}{M_p^2} \sim \frac{|W_h|}{M_p^2} \sim \Lambda , \quad (4.11)$$

of the correct order of magnitude.

4.2 Soft breaking terms

Supersymmetry breaking is communicated to the matter fields in the visible brane by gravitational mediation. The resulting soft masses will be given in terms of the two (complex) quantities: W_h and $\widetilde{W}_v \equiv W_v \exp[-3\langle T \rangle]$. These two sources of SUSY breaking are related by the minimization condition (4.7), so that if we write $W_h = |W_h|e^{i\gamma}$, then $\widetilde{W}_v = -|\widetilde{W}_v|e^{i\gamma}$. All soft masses can then be given in terms of the complex mass

$$\tilde{m}_{3/2} \equiv m_{3/2} e^{i\gamma} . \quad (4.12)$$

Although $m_{3/2}$ is dominated by the first source of SUSY breaking (W_h), in general both sources can give comparable contributions to other soft masses.

The effects of soft SUSY breaking in masses and other couplings of scalar observable fields, φ_i , are obtained from the SUGRA effective potential (4.2) where now we include also matter fields in the sum, i.e. $I, J = \varphi_i, T, S$, and use the complete effective superpotential

$$\begin{aligned} W &= e^{\langle K/2M_p^2 \rangle} W_{sbr} + W_{\text{eff}} \\ &= \tilde{m}_{3/2} \left[M_p^2 - \frac{1}{3} \Lambda^2 e^{-3t/\Lambda} \right] \frac{f(S)}{f(\langle S \rangle)} + W_{\text{eff}} , \end{aligned} \quad (4.13)$$

where W_{eff} is the effective superpotential for the matter fields, given in (3.5), and we have made the shift $T = \langle T \rangle + t/\Lambda$ to the physical field t . The corresponding shift for

S is $S = \langle S \rangle + 2s\langle \text{Re}S \rangle/M_p$. It turns out that the interactions of the shifted field s to matter fields are always suppressed by inverse powers of M_p . This is, essentially, a consequence of its normalization factor, $\sim M_p^{-1}$ (note that the normalization factor for t is $\sim \Lambda^{-1}$). Hence, we concentrate here in the soft-terms for $\{\phi_I\} = \{\varphi_i, t\}$ only. We can conveniently write these contributions in terms of K_{eff} , Σ_{eff} and W_{eff} as defined in eqs. (3.3–3.5). Making an expansion in powers of M_p^{-1} we find, for the L_J 's defined in (4.3):

$$L_S = (\varsigma - 1)\tilde{m}_{3/2} \left[M_p - \frac{1}{3M_p} \Lambda^2 e^{-3t/\Lambda} \right] - \frac{W_{\text{eff}}}{M_p} + \dots \quad (4.14)$$

$$L_t = \tilde{m}_{3/2} \Lambda \left[e^{-3t/\Lambda} - \frac{K_{\text{eff}}}{\Lambda^2} \right] - 3 \frac{W_{\text{eff}}}{\Lambda} + \dots \quad (4.15)$$

$$L_{\varphi_i} = \frac{\partial W_{\text{eff}}}{\partial \varphi_i} + \frac{\tilde{m}_{3/2}}{\Lambda^2} K_{\text{eff}} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} + \dots \quad (4.16)$$

Notice that, prior to electroweak breaking, $\langle L_S \rangle = (\varsigma - 1)\tilde{m}_{3/2}M_p$ and $\langle L_t \rangle = \langle L_{\varphi_i} \rangle = 0$, corresponding to the fact that it is F_S that breaks SUSY. After substituting (4.14–4.16) in the SUGRA potential, keeping only terms not suppressed by M_p^{-1} , this can be expressed in the following suggestive form,

$$V_{\text{SUGRA}} = \mathcal{G}_{IJ}^{-1} \left(\frac{\partial \mathcal{W}_{\text{eff}}}{\partial \phi_I} \right)^* \frac{\partial \mathcal{W}_{\text{eff}}}{\partial \phi_J} - \left[(2 + \varsigma)\tilde{m}_{3/2}^* W_{\text{eff}} + \text{h.c.} \right], \quad (4.17)$$

where we have introduced the non-holomorphic function

$$\mathcal{W}_{\text{eff}} \equiv W_{\text{eff}} - \frac{1}{3}\tilde{m}_{3/2}\Lambda^2 e^{-3t/\Lambda} + \tilde{m}_{3/2}K_{\text{eff}}. \quad (4.18)$$

The first term in the right hand side of (4.17) is exactly of the form of a SUSY potential. However, \mathcal{W}_{eff} , being non-holomorphic, cannot be interpreted as a superpotential. Nevertheless, it is instructive to identify the holomorphic pieces of \mathcal{W}_{eff} which may be interpreted as an effective superpotential W'_{eff} . In an expansion in powers of Λ^{-1} , keeping only renormalizable terms, we get

$$\begin{aligned} W'_{\text{eff}} &= \frac{2}{3}\tilde{m}_{3/2}\Lambda^2 + (\mu_0 + \lambda\tilde{m}_{3/2})H_1 \cdot H_2 + h_U^{ij}Q_{Li} \cdot H_2 U_{Rj} + h_D^{ij}H_1 \cdot Q_{Li} D_{Rj} \\ &- \tilde{m}_{3/2}\Lambda t^2 - \frac{1}{\Lambda}(3\mu_0 + \lambda\tilde{m}_{3/2})tH_1 \cdot H_2 + \frac{4}{3}\frac{\tilde{m}_{3/2}}{\Lambda}t^3. \end{aligned} \quad (4.19)$$

This corresponds to a NMSSM superpotential, where the t field plays the role of the NMSSM singlet, with a non-zero μ -term (while the usual NMSSM assumes it is absent to start with and is generated dynamically by the singlet VEV). However, the model

cannot be interpreted as a particular version of the NMSSM for several reasons: the Kähler metric \mathcal{G}_{IJ} in eq.(4.17) derives from a non-minimal Kähler potential, K_{eff} ; and, besides the “supersymmetric” part, eq.(4.17) contains extra pieces [the last term plus the non-holomorphic pieces in \mathcal{W}_{eff} , which are not accounted for by (4.19)], which correspond to SUSY breaking terms. These terms are more general than those usually considered for the NMSSM (*e.g.* SUSY-breaking quartic Higgs couplings are present). Consequently, the expected phenomenology will be completely different.

Let us stress that eq.(4.19) shows explicitly the two sources for the μ parameter:

$$\mu \equiv \mu_0 + \lambda \tilde{m}_{3/2}. \quad (4.20)$$

One is simply the previously mentioned supersymmetric Higgs mass of order $\mathcal{O}(M_p)$ in the observable superpotential, which is effectively reduced to $\mathcal{O}(\Lambda)$ by the warp factor; the other is the $[\lambda H_1 \cdot H_2 + \text{h.c.}]$ term in the Kähler potential (3.3) that, after SUSY breaking, contributes to the μ parameter via the Giudice-Masiero [25] mechanism.

We can simplify the potential (4.17) by using the explicit form of \mathcal{G}_{IJ}^{-1} given in (3.12). We find soft terms in the scalar potential proportional to $m_{3/2}^2$ of the form

$$\begin{aligned} \delta V_{\text{soft}} &= m_{3/2}^2 \left\{ \frac{\Lambda^2}{K_{\text{eff}}} \left[\Lambda^2 + \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i^*} \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j^*} \right]^{-1} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_j} \right] e^{-3(t+t^*)/\Lambda} \right. \\ &\quad \left. - \Lambda^2 \left[e^{-3t/\Lambda} + e^{-3t^*/\Lambda} \right] + K_{\text{eff}} \right\}. \end{aligned} \quad (4.21)$$

plus soft terms which are linear in the gravitino mass:

$$\begin{aligned} \delta V_{\text{soft}} &= \tilde{m}_{3/2} \left\{ \frac{\Lambda^2}{K_{\text{eff}}} \left(\frac{\partial W_{\text{eff}}}{\partial \varphi_i} - \frac{3W_{\text{eff}}}{\Lambda^2} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} \right)^* \left[\frac{\partial^2 \Sigma_{\text{eff}}}{\partial \varphi_i \partial \varphi_j^*} \right]^{-1} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_j} e^{-3t/\Lambda} \right. \\ &\quad \left. + W_{\text{eff}}^* \left[1 - \varsigma - \frac{3\Lambda^2}{K_{\text{eff}}} e^{-3t/\Lambda} \right] \right\} + \text{h.c.} \end{aligned} \quad (4.22)$$

An expansion of (4.21) and (4.22) in powers of φ_i/Λ and t/Λ makes manifest the properties of the SUSY breaking terms in this scenario. In increasing order of field powers, we find:

- a) vanishing field-independent part of V_{soft} .
- b) vanishing t -tadpole.
- c) universal soft mass (equal to $m_{3/2}$) for the scalar fields φ_i 's.

- d) the real part of the radion field has a soft mass $m_{3/2}$ while its imaginary part has mass $3m_{3/2}$.
- e) B -type and A -type soft terms. More precisely, the A -parameters are universal, $A^* = \tilde{m}_{3/2}(1 - \varsigma)$, while $B\mu = (-\varsigma^* \tilde{m}_{3/2}^*)\mu_0 + (2\tilde{m}_{3/2}^*)\lambda\tilde{m}_{3/2}$, where ς and μ are given by eqs.(4.9, 4.20). The latter expression reflects the two different contributions to the B -parameter, associated with the two sources of μ .
- f) trilinear terms like $t^*|\varphi|^2 + \text{h.c.}$, where φ is any chiral scalar field. These terms always involve the radion field, t .
- g) dimension $4+n$ SUSY-breaking operators suppressed only by powers of $m_{3/2}/\Lambda^{1+n}$.

The points a), b) above are a consequence of our previous assumption of vanishing cosmological constant and the fact that we have defined t as the physical field that describes fluctuations of T around its tree-level minimum $\langle T \rangle$ (arising from $F_T = 0$). The other points are more or less conventional, except for points f) and g). Actually, one may worry about the quadratic divergences introduced by the dimension $D \geq 4$ operators, with undesirable consequences for the gauge-hierarchy. Certainly, non-supersymmetric $D = 4$ operators give a non-zero contribution to the field-independent part of $\text{Str}\mathcal{M}^2$, and thus to the quadratically-divergent part of the one-loop effective potential

$$V_{1\text{-loop}}^{\text{quad}} = \frac{1}{32\pi^2} \Lambda_{UV}^2 \text{Str}\mathcal{M}^2, \quad (4.23)$$

where Λ_{UV} is the ultraviolet cut-off of the effective theory. As discussed in sect. 2, $\Lambda_{UV} \simeq \Lambda = \mathcal{O}(\text{TeV})$. One can think that, provided Λ is not much bigger than 1 TeV, the contribution (4.23) will be small enough not to spoil the naturalness of the electroweak breaking. But, actually, even for $\Lambda \gg 1$ TeV the contribution to $V_{1\text{-loop}}^{\text{quad}}$ remains under control. The reason is that, for a renormalizable W_{eff} , all the $D \geq 4$ operators that contribute to (4.23) contain at least a factor Λ^{-2} that precisely compensates the Λ_{UV}^2 factor⁸ (incidentally, this also occurs for the supersymmetric $D \geq 4$ operators of sect. 2). Then, provided $m_{3/2}$ and μ are of order $\mathcal{O}(\text{TeV})$, $V_{1\text{-loop}}^{\text{quad}}$ is small, thanks to the additional suppression provided by the 1-loop $1/32\pi^2$ factor.

⁸If W_{eff} contains non-renormalizable operators, like the one in eq. (3.21), this is no longer true. Such a term in the superpotential induces a term $\sim (m_{3/2}/\Lambda)\phi^4$ after SUSY breaking. However this term is still harmless, as it gives a vanishing contribution to $\text{Str}\mathcal{M}^2$ (in this sense it is a “soft” term).

Soft gaugino masses are also generated and given by

$$M_A = \frac{1}{2} F_i \frac{\partial \text{Re} f_A}{\partial \phi_i} , \quad (4.24)$$

(A is a gauge index). In our case, SUSY is broken along the S direction, so that M_A will be sizeable [$\sim \mathcal{O}(\Lambda)$] as long as the gauge kinetic functions, f_A , have a non-trivial S -dependence. Of course, this is so in the most ordinary case, i.e. when S represents the dilaton and $f_A \sim S$. Even if one departs from this simple situation (and we argued in favor of such a case in sect. 2), f_A will generically present a non-trivial S -dependence, thus guaranteeing sizeable gaugino masses. *E.g.* if S represents a modulus, U , parameterizing one of the three complex extra dimensions, f_A may have a non-trivial dependence on U . This occurs in the context of M -theory compactified à la Horava-Witten [26] and for appropriate 5-branes in the context of Type I constructions [27]. It may also occur in the weakly-coupled heterotic string if the threshold corrections are large. We prefer to leave M_A as a free parameter since its precise value depends on this kind of details. In fact, in many other constructions f_A has important contributions which are model- and gauge-group-dependent, and prevent the writing of a general expression for f_A (*e.g.* in Type I constructions there are model-dependent contributions proportional to the twisted moduli).

Finally, we discuss the effects of SUSY breaking for the masses of matter fermions $\chi_I = \{\chi_i, \chi_t\}$. From the SUGRA Lagrangian we obtain (after eliminating the mixing between matter fermions and the gravitino [10]):

$$\begin{aligned} \delta \mathcal{L}_{\text{SUGRA}} = & \frac{1}{2} \exp[K/(2M_p^2)] \frac{|W|}{W} \left\{ \frac{\partial L_J}{\partial \phi_I} - \frac{\partial^3 K}{\partial \phi_I \partial \phi_J \partial \phi_L^*} \mathcal{G}_{LK}^{-1} L_K \right. \\ & \left. + \frac{1}{W} \left[-\frac{\partial W}{\partial \phi_I} L_J + \frac{1}{3} L_I L_J \right] \right\} (\chi_I \cdot \chi_J) + \text{h.c.} \end{aligned} \quad (4.25)$$

where L_I was defined in (4.3). Substituting the L_I expansions as given in (4.14–4.16) we arrive at the $M_p \rightarrow \infty$ limit

$$\delta \mathcal{L}_{\text{SUGRA}} \rightarrow \frac{1}{2} \left\{ -\frac{\partial^2 \mathcal{W}_{\text{eff}}}{\partial \phi_I \partial \phi_J} + \frac{\partial^3 K_{\text{eff}}}{\partial \phi_I \partial \phi_J \partial \phi_L^*} \mathcal{G}_{LK}^{-1} \frac{\partial \mathcal{W}_{\text{eff}}}{\partial \phi_K} \right\} (\chi_I \cdot \chi_J) + \text{h.c.} , \quad (4.26)$$

with \mathcal{W}_{eff} as defined in (4.18). We see once again that (4.26) has a supersymmetric form [compare with (3.36)] but is not supersymmetric because \mathcal{W}_{eff} is a non-holomorphic function of the chiral fields.

Rewriting (4.26) in terms of Σ_{eff} using (3.12) and explicitly replacing $\phi_I = \{\varphi_i, t\}$, $\chi_I = \{\chi_i, \chi_t\}$, we obtain the following $\tilde{m}_{3/2}$ -dependent terms:

$$\begin{aligned} \delta\mathcal{L}_{\text{soft}} = & \frac{1}{2}\tilde{m}_{3/2}e^{-3t/\Lambda} \left\{ \left[\frac{1}{\Lambda^2} \frac{\partial\Sigma_{\text{eff}}}{\partial\varphi_i} \frac{\partial\Sigma_{\text{eff}}}{\partial\varphi_j} - \frac{\partial^2\Sigma_{\text{eff}}}{\partial\varphi_i\partial\varphi_j} \right. \right. \\ & \left. \left. + \frac{\partial^3\Sigma_{\text{eff}}}{\partial\varphi_i\partial\varphi_j\partial\varphi_l^*} \left[\frac{\partial^2\Sigma_{\text{eff}}}{\partial\varphi_l\partial\varphi_k^*} \right]^{-1} \frac{\partial\Sigma_{\text{eff}}}{\partial\varphi_k} \right] (\chi_i \cdot \chi_j) + 2(\chi_t \cdot \chi_t) \right\} + \text{h.c.} \quad (4.27) \end{aligned}$$

All the expressions of this subsection have been derived from the mechanism of SUSY breaking discussed in the previous subsection, i.e. an S -dependent W_{subr} superpotential and $K(S, S^\dagger) = -M_p^2 \log(S + S^\dagger)$. In the alternative no-scale scenario, with $K(S, S^\dagger) = -3M_p^2 \log(S + S^\dagger)$ and S -independent W_{subr} , SUSY is also broken with vanishing tree-level cosmological constant, as discussed in the previous subsection. Then the results for the soft breaking terms would be identical to those obtained in this subsection, with the simple replacement $\varsigma \rightarrow -2$ in all expressions [compare to the previous definition in eq.(4.9)].

5 Electroweak Symmetry breaking

The starting point to analyze the electroweak breaking is the complete Higgs potential, which can be obtained adding to the SUSY potential (3.17) the soft terms given by eqs. (4.21) and (4.22). Explicitly, the mass terms read

$$\begin{aligned} V_2 = & (|\mu|^2 + m_{3/2}^2)(|H_1|^2 + |H_2|^2) - m_{3/2}^2[2(t^2 + t^{*2}) - 5tt^*] \\ & + \left\{ \tilde{m}_{3/2}^* \left[2\lambda\tilde{m}_{3/2} - \varsigma^*\mu_0 \right] H_1 \cdot H_2 + \text{h.c.} \right\} , \quad (5.1) \end{aligned}$$

In general, the two contributions to the μ parameter, $\mu = \mu_0 + \lambda\tilde{m}_{3/2}$, should be considered. If one chooses $\mu_0 = 0$ (which is respected by radiative corrections due to SUSY non-renormalization theorems) one still can get an acceptable electroweak breaking, with $\lambda\tilde{m}_{3/2}$ as only source for μ . On the other hand, the option $\lambda = 0$ and $\mu = \mu_0$ is problematic for phenomenology as is discussed below. Note also that the condition $\lambda = 0$ is not protected by non-renormalization theorems so that a non-zero λ could be generated by radiative corrections (provided it is not protected by a Peccei-Quinn symmetry).

To discuss the breaking of the electroweak symmetry we need the rest of the Higgs effective potential, V . Provided v/Λ is small, it is enough to keep in V up to quartic

terms in the Higgs fields (higher order corrections might be included when necessary). The cubic terms in the potential are:

$$V_3 = 3\frac{m_{3/2}^2}{\Lambda}(t+t^*)\left[(t+t^*)^2 - \frac{9}{2}tt^*\right] - \frac{1}{\Lambda}(m_{3/2}^2 + 2|\mu|^2)(t+t^*)(|H_1|^2 + |H_2|^2) + \left\{\frac{\tilde{m}_{3/2}^*}{\Lambda}\left[3(\varsigma^* - 1)\mu_0 t + (2\mu_0 - 3\lambda\tilde{m}_{3/2})(t+t^*)\right]H_1 \cdot H_2 + \text{h.c.}\right\}. \quad (5.2)$$

From this expression we see that non-zero values for the Higgs doublets generate a tadpole for the radion, so that the field t also acquires a VEV as a consequence of electroweak symmetry breaking. This VEV is fixed so as to cancel the t -tadpole and represents a small correction [of $\mathcal{O}(v^2/\Lambda^2)$] to the VEV of the original radion field T . As such it can be absorbed in a small redefinition of the scale Λ .

Finally, the quartic terms of the potential are:

$$V_4 = \frac{m_{3/2}^2}{\Lambda^2}\left[\frac{17}{24}(t+t^*)^4 - \frac{27}{8}(t^4 + t^{*4})\right] + \left[\frac{m_{3/2}^2}{2\Lambda^2} + 2\frac{|\mu|^2}{\Lambda^2}\right](t+t^*)^2(|H_1|^2 + |H_2|^2) + \left\{\frac{\tilde{m}_{3/2}^*}{2\Lambda^2}\left[9(1 - \varsigma^*)\mu_0 t^2 + (5\lambda\tilde{m}_{3/2} - 4\mu_0)(t+t^*)^2\right]H_1 \cdot H_2 + \text{h.c.}\right\} + \left[\frac{g^2}{8} - \frac{|\mu|^2}{\Lambda^2}\right](|H_1|^2 + |H_2|^2)^2 + \frac{g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \left\{-\frac{g^2}{2} + \frac{1}{\Lambda^2}\left[5|\lambda\tilde{m}_{3/2}|^2 - 3|\mu|^2 - 2(\lambda\tilde{m}_{3/2}\mu^* + \text{h.c.})\right]\right\}|H_1 \cdot H_2|^2 + \frac{1}{\Lambda^2}\left[\lambda\tilde{m}_{3/2}^*(4\lambda\tilde{m}_{3/2} - 5\mu)(H_1 \cdot H_2)^2 + \text{h.c.}\right] + \frac{1}{\Lambda^2}\left[(\lambda m_{3/2}^2 - 2\mu\tilde{m}_{3/2}^* + 3\lambda^2\mu^*\tilde{m}_{3/2} - 4\lambda|\mu|^2)H_1 \cdot H_2 + \text{h.c.}\right](|H_1|^2 + |H_2|^2)^2. \quad (5.3)$$

There are significant differences in this potential with respect to the case of the MSSM (or the NMSSM). It contains new couplings, both supersymmetric and non-supersymmetric, discussed already in the two previous sections. The importance of such terms is manifold, as we are about to see. In particular, they turn out to completely modify the usual pattern of symmetry breaking, which allows for a Higgs mass, M_{h^0} , much larger than in the MSSM. If h^0 is discovered, knowledge of its mass (*e.g.* if the LEP2 excess is confirmed and $M_{h^0} \sim 115.6$ GeV [28]) would give information on μ_0/Λ or $m_{3/2}/\Lambda$.

Another important difference from the MSSM is that the couplings and masses in $V(=V_2 + V_3 + V_4)$ as written above are assumed to be evaluated at the scale Λ , which is close to the electroweak scale. As a result, the evolution of these parameters from Λ

down to the typical scale of SUSY masses ($\sim m_{3/2}$, the relevant scale for the study of electroweak symmetry breaking) gives in general a small correction. This means that the breaking of the electroweak symmetry is not a radiative effect, like in the MSSM, but has to occur at tree-level⁹.

In the MSSM, with mass terms $V_2 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_{12}^2 (H_1 \cdot H_2 + \text{h.c.})$, a proper electroweak breaking requires, first, that the origin is destabilized, or

$$m_1^2 m_2^2 - m_{12}^4 < 0 , \quad (5.4)$$

and second, that the potential is not unbounded from below along the D -flat direction $H_1^0 = H_2^0$, which requires a positive mass term along such direction:

$$m_1^2 + m_2^2 + 2m_{12}^2 > 0 . \quad (5.5)$$

In the MSSM, before radiative corrections, conditions (5.4, 5.5) are incompatible if the Higgs masses, m_1^2 and m_2^2 , are degenerate (as it happens here). However, in our case, condition (5.5) is no longer necessary because now there is a non-zero quartic coupling along the direction $H_1^0 = H_2^0$, which is no longer flat.

Looking at the neutral components of the Higgs doublets, $H_j^0 \equiv (h_j^{0r} + i h_j^{0i})/\sqrt{2}$, with $j = 1, 2$, the minimization conditions read

$$\frac{\partial V}{\partial h_1^{0r}} = 0 , \quad \frac{\partial V}{\partial h_2^{0r}} = 0 , \quad (5.6)$$

from which we can get the Higgs VEVs, or alternatively $v^2 \equiv \langle h_1^{0r} \rangle^2 + \langle h_2^{0r} \rangle^2$ and $\tan \beta \equiv \langle h_2^{0r} \rangle / \langle h_1^{0r} \rangle$, as functions of the parameters in V . Assuming for simplicity that all the parameters in V are real, we obtain, for the VEV v :

$$v^2 = \frac{-m_\beta^2}{\lambda_\beta} , \quad (5.7)$$

with

$$m_\beta^2 \equiv (\mu^2 + m_{3/2}^2) + [\lambda(2 + \varsigma)m_{3/2}^2 - \varsigma\mu m_{3/2}] \sin 2\beta , \quad (5.8)$$

$$\begin{aligned} \lambda_\beta \equiv & \frac{1}{8}(g^2 + g'^2) \cos^2 2\beta - \frac{\mu^2}{\Lambda^2} \left[1 + \frac{3}{4} \sin^2 2\beta + 4\lambda \sin 2\beta \right] \\ & + \frac{m_{3/2}^2}{\Lambda^2} \left[\frac{13}{4} \lambda^2 \sin^2 2\beta + \lambda \sin 2\beta \right] + \frac{\mu m_{3/2}}{\Lambda^2} \left[-\frac{7}{2} \lambda \sin^2 2\beta + (3\lambda^2 - 2) \sin 2\beta \right] , \end{aligned} \quad (5.9)$$

⁹Incidentally, for the very same reason we do not expect sizeable radiatively-generated off-diagonal entries in the squark and slepton mass matrices. Hence, these models are very safe regarding FCNC effects.

and, for $\tan \beta$:

$$0 = \cos 2\beta \left\{ 4 \left[\lambda(v^2 + 2(2 + \varsigma)\Lambda^2)m_{3/2}^2 + [(3\lambda^2 - 2)v^2 - 2\varsigma\Lambda^2]\mu m_{3/2} - 4\lambda v^2 \mu^2 \right] - v^2 \left[(g^2 + g'^2)\Lambda^2 - 26\lambda^2 m_{3/2}^2 + 28\lambda\mu m_{3/2} + 6\mu^2 \right] \sin 2\beta \right\} . \quad (5.10)$$

The interpretation of (5.7) is straightforward: in the direction of field space, $\varphi \equiv h_1^{0r} \cos \beta + h_2^{0r} \sin \beta$, along which the minimum lies (by definition of $\tan \beta$), the potential is simply

$$V(\varphi) = \frac{1}{2}m_\beta^2 \varphi^2 + \frac{1}{4}\lambda_\beta \varphi^4 , \quad (5.11)$$

and minimization of this potential leads directly to eq. (5.7). A correct breaking clearly requires

$$m_\beta^2 < 0 , \quad (5.12)$$

$$\lambda_\beta > 0 . \quad (5.13)$$

Condition (5.12) is equivalent to the MSSM eq. (5.4), while condition (5.13) replaces eq. (5.5). Before analyzing whether these two conditions can be satisfied, we turn to eq. (5.10), from which we can determine $\tan \beta$. This equation can be satisfied either by

$$\tan \beta = 1 , \quad (5.14)$$

or by

$$\sin 2\beta = \frac{4 \left[\lambda(v^2 + 2(2 + \varsigma)\Lambda^2)m_{3/2}^2 + [(3\lambda^2 - 2)v^2 - 2\varsigma\Lambda^2]\mu m_{3/2} - 4\lambda v^2 \mu^2 \right]}{v^2 \left[(g^2 + g'^2)\Lambda^2 - 26\lambda^2 m_{3/2}^2 + 28\lambda\mu m_{3/2} + 6\mu^2 \right]} . \quad (5.15)$$

This second solution, however, cannot be accepted because it leads to problems. To get $\sin 2\beta < 1$ there must be a cancellation in the numerator of (5.15), which is otherwise of order $\sim \Lambda^4$ (taking $m_{3/2}$ and μ not far from Λ) while the denominator is only of order $\sim \Lambda^2 v^2$. The cancellation must be $\lambda(2 + \varsigma)m_{3/2}^2 - \varsigma\mu m_{3/2} \sim \mathcal{O}(v^2)$, but, looking at (5.8), it is clear that this makes the condition $m_\beta^2 < 0$ impossible to satisfy. Another, more indirect, way of seeing that solution (5.15) is not acceptable is that it leads to a tachyonic charged Higgs with mass (squared), $M_{H^\pm}^2 = -\frac{1}{4}g'^2 v^2$.

We are therefore forced to choose $\tan \beta = 1$ and, using this in (5.7) and (5.10), the requirements in (5.12, 5.13) read

$$m_{\beta=\pi/4}^2 = (\mu^2 + m_{3/2}^2) + \left[\lambda(2 + \varsigma)m_{3/2}^2 - \varsigma\mu m_{3/2} \right] < 0 , \quad (5.16)$$

$$\lambda_{\beta=\pi/4} = \left(\frac{13}{4}\lambda^2 + \lambda \right) \frac{m_{3/2}^2}{\Lambda^2} + \left(3\lambda^2 - \frac{7}{2}\lambda - 2 \right) \frac{\mu m_{3/2}}{\Lambda^2} - \left(4\lambda + \frac{7}{4} \right) \frac{\mu^2}{\Lambda^2} > 0 . \quad (5.17)$$

We see that the coupling λ plays a crucial role in electroweak symmetry breaking: if $\lambda = 0$, then $m_\beta^2 < 0$ is easy to satisfy, provided $(2 + \varsigma)\mu m_{3/2} > (\mu + m_{3/2})^2 \geq 0$, which requires $\mu m_{3/2} > 0$. In that case, we obtain $\lambda_\beta = -[2\mu m_{3/2} - 7\mu^2/4]/\Lambda^2 < 0$, which contradicts the requirement (5.13). Instead, by choosing $\lambda \neq 0$ conveniently, the two conditions can be satisfied simultaneously. In order to get $v^2 = -m_\beta^2/\lambda_\beta \sim (246 \text{ GeV})^2$, with λ_β perturbative, there must be a cancellation in (5.16) so as to give $m_\beta \sim \mathcal{O}(v)$ [or, more precisely $m_\beta \sim \mathcal{O}(\lambda_\beta^{1/2} v)$]. This cancellation requires a tuning similar to that in the MSSM (or even milder, since λ_β is normally bigger than in the MSSM). As we are interested in a situation with $v/\Lambda \ll 1$, this cancellation requires

$$\lambda \simeq \frac{-1}{(2 + \varsigma)} \left[1 - \varsigma \frac{\mu}{m_{3/2}} + \frac{\mu^2}{m_{3/2}^2} \right] + \mathcal{O}\left(\frac{v^2}{\Lambda^2}\right). \quad (5.18)$$

This is a parabola in the variable $\mu/m_{3/2}$. After substitution of this expression in (5.17), λ_β is a polynomial of fifth degree in $\mu/m_{3/2}$. Therefore, it is guaranteed that λ_β takes perturbatively small and positive values at least near one root of that polynomial.

The reason why $\tan \beta = 1$ is a solution of the minimization conditions is related to the fact that the potential for the neutral Higgs components is symmetric under $H_1^0 \leftrightarrow H_2^0$. In fact, for $g' = 0$ the potential has a larger symmetry because it is a function of H_1 and H_2 only through the combinations

$$\text{Tr} [\mathbf{H}^\dagger \mathbf{H}] = |H_1|^2 + |H_2|^2, \quad (5.19)$$

$$\text{Det} [\mathbf{H}] = H_1 \cdot H_2, \quad (5.20)$$

where \mathbf{H} is defined in (3.30). This means that V is approximately invariant under the $SU(2)_L \times SU(2)_R$ symmetry discussed in connection to $\Delta\rho$ in section 3. For general $\tan \beta$, the spontaneous breaking of this symmetry leads to three Goldstone bosons:

$$\begin{aligned} G^0 &\equiv \text{Im}(H_1^0) \cos \beta - \text{Im}(H_2^0) \sin \beta, \\ G^+ &\equiv H_1^+ \cos \beta - H_2^+ \sin \beta, \\ P^+ &\equiv H_1^+ \sin \beta - H_2^+ \cos \beta. \end{aligned} \quad (5.21)$$

Here, G^+ and G^0 are the true Goldstone bosons eaten up by W^+ and Z^0 , while P^+ is a pseudo-Goldstone. Non-zero g' introduces a small explicit breaking of the symmetry and P^+ acquires a mass

$$M_{P^+}^2 = \langle P^- | \mathbf{M}_{\text{ch}}^2 | P^+ \rangle = -\frac{1}{4} g'^2 v^2 \cos^2 2\beta, \quad (5.22)$$

(where \mathbf{M}_{ch}^2 is the charged Higgs mass matrix). Although P^+ is not an eigenstate of \mathbf{M}_{ch}^2 , $M_{P^+}^2 < 0$ implies a negative mass eigenvalue, which corresponds to the charged Higgs mass $M_{H^\pm}^2 = -\frac{1}{4}g'^2 v^2$ mentioned before. This means that $\tan\beta \neq 1$ is not a true minimum of the potential. This problem disappears for $\tan\beta = 1$, case in which $P^+ \equiv G^+$, $M_{P^+}^2 = 0$ and the charged Higgs mass is not even suppressed by g' .

In general, besides the g' -terms, Yukawa couplings also break the custodial symmetry. As a consequence, $\tan\beta = 1$ is not protected from radiative corrections (like those arising from the short running between Λ and $m_{3/2}$) and we expect some small deviations from $\tan\beta = 1$. With this additional breaking of the custodial symmetry, the mass of the pseudogoldstone boson [eq. (5.22)] receives new contributions and can lead to $M_{P^+}^2 > 0$, so that $\tan\beta$ close to (but different from) 1 is not problematic.

Hence, we consider $\tan\beta \simeq 1$ as a quite robust prediction of SUSY warped scenarios with universal Higgs masses. Actually, for other choices of the Kähler potential we also expect $\tan\beta \simeq 1$, as it is explicitly illustrated in Appendix B.

6 Higgs sector

In the MSSM, $\tan\beta = 1$ corresponds to a D -flat direction in the tree-level Higgs potential. The tree-level Higgs mass is zero along that direction and, although radiative corrections can give a large contribution to this mass, the experimental limit from LEP2 is able to exclude the interval $0.5 \lesssim \tan\beta \lesssim 2.4$ at 95% C.L. [29]. In our model the situation is completely different because, in addition to the D -term contribution to λ_β (which is still zero for $\beta = \pi/4$) there are new corrections which can make $\lambda_\beta > 0$ already at tree-level. As we show later on, the Higgs mass can easily evade LEP2 limits.

After electroweak symmetry breaking, out of the ten initial degrees of freedom (d.o.f.'s) in H_1 , H_2 and t , three d.o.f.'s [G^\pm and G^0 in (5.21)] are absorbed in the longitudinal components of the massive gauge bosons, W^\pm and Z^0 . The mass eigenstates corresponding to the seven remaining d.o.f.'s appear as three \mathcal{CP} -even scalars (h^0, h'^0, H^0), two \mathcal{CP} -odd pseudoscalars (A^0, A'^0) and one charged Higgs (H^\pm). The calculation of the masses of these physical Higgses requires two steps. First, one obtains the corresponding mass matrices as the second derivatives of the effective potential evaluated at the electroweak minimum. Then one must take into account that a

non-minimal Kähler potential leads, after symmetry breaking, to non-canonical kinetic terms for scalar fields (see the discussion in subsection 3.2). The kinetic terms can be recast in canonical form by a suitable (non-unitary) field redefinition and the mass matrices have to be re-expressed in the basis of the canonically-normalized fields. The final masses are the eigenvalues of these transformed mass matrices. The details of this procedure are given in Appendix C. Here we summarize the results.

In the \mathcal{CP} -even sector, using $\{\varphi^0 \equiv (h_1^{0r} + h_2^{0r})/\sqrt{2}, H^0 \equiv (h_1^{0r} - h_2^{0r})/\sqrt{2}, t_R^0\}$, (with $h_{1,2}^{0r} \equiv \sqrt{2}\text{Re}[H_{1,2}^0]$ and $t_R^0 \equiv \sqrt{2}\text{Re}[t]$) as a convenient basis, we find that H^0 is a mass-eigenstate, with $M_{H^0}^2$ controlled by $\mu m_{3/2}$ and $m_{3/2}^2$ [see (C.12)], while φ^0 and t_R^0 are in general mixed to give the two mass eigenstates h^0 and h'^0 . The exact masses of h^0 and h'^0 can be simply obtained as the eigenvalues of a 2×2 matrix, but the resulting expressions are not very illuminating. In this respect it is useful to note that the mass of the lightest \mathcal{CP} -even Higgs, h^0 , satisfies the mass bound (\mathbf{M}_h^2 is the \mathcal{CP} -even mass matrix)

$$M_{h^0}^2 \leq \langle \varphi^0 | \mathbf{M}_h^2 | \varphi^0 \rangle = 2(\lambda_\beta + \xi_\beta)v^2, \quad (6.1)$$

$$M_{h^0}^2 \leq \langle t_R^0 | \mathbf{M}_h^2 | t_R^0 \rangle, \quad (6.2)$$

with λ_β defined in (5.10) and ξ_β , which represents a shift in the quartic coupling λ_β due to kinetic mixing between $H_{1,2}^0$ and t , is given in Appendix C. The virtue of this bound, eq. (6.1), is that it is controlled by the electroweak scale v : it can only be made heavy at the expense of making the coupling $(\lambda_\beta + \xi_\beta)$ strong. The existence of such bound follows from general arguments (see *e.g.* [30] and references therein). On the other hand, as we explicitly show below, the lightest \mathcal{CP} -even Higgs boson has in general some singlet component coming from t_R^0 and this makes the couplings of h^0 different from the SM ones.

In the \mathcal{CP} -odd sector, we use the basis $\{G^0 \equiv (h_1^{0i} - h_2^{0i})/\sqrt{2}, a^0 \equiv (h_1^{0i} + h_2^{0i})/\sqrt{2}, t_I^0\}$ (where $h_{1,2}^{0i} \equiv \sqrt{2}\text{Im}[H_{1,2}^0]$ and $t_I^0 \equiv \sqrt{2}\text{Im}[t]$). The neutral Goldstone boson, G^0 , is an exact eigenstate, with zero mass, while in general a^0 and t_I^0 are mixed to give the two mass eigenstates A^0 and A'^0 . From the expressions in Appendix C one can see that, for μ and $m_{3/2}$ moderately larger than v , a^0 and t_I^0 are approximate mass eigenstates up to $\mathcal{O}(v/\Lambda)$ corrections: $A^0 \simeq a^0$ and $A'^0 \simeq t_I^0$.

In the charged Higgs sector we find that the charged Goldstone boson is $G^+ \equiv (H_1^+ - H_2^+)/\sqrt{2}$ and the physical Higgs $H^+ \equiv (H_1^+ + H_2^+)/\sqrt{2}$. The mass for this

charged Higgs boson is

$$M_{H^\pm}^2 = M_{H^0}^2 - \frac{1}{4}g'^2 v^2, \quad (6.3)$$

with $M_{H^0}^2$ as defined in eq. (C.12).

The pattern of Higgs masses just described conforms neatly with the requirements of the underlying symmetries of the Higgs potential, V . We have already shown that, after electroweak symmetry breaking with $\tan\beta = 1$, V still has an approximate $SU(2)_{L+R}$ custodial symmetry only broken by small g' -terms. Therefore, the physical Higgs fields belong in representations of $SU(2)_{L+R}$, with the members of the same multiplet having degenerate masses [up to small $\mathcal{O}(g'^2 v^2)$ splittings]. To show how this happens, notice first that a real $SU(2)_{L+R}$ triplet, $\{\xi^+, \xi^0, \xi^-\}$, can be written as the 2×2 matrix

$$\mathbf{T}_\xi = \frac{1}{\sqrt{2}} \boldsymbol{\sigma}^a \xi_a = \begin{bmatrix} \xi^0/\sqrt{2} & \xi^+ \\ \xi^- & -\xi^0/\sqrt{2} \end{bmatrix}, \quad (6.4)$$

with $\xi^0 \equiv \xi_3$ and $\xi^\pm \equiv (\xi_1 \mp i\xi_2)/\sqrt{2}$. The transformation law $\mathbf{T}_\xi \rightarrow \mathbf{U}_{\mathbf{L+R}} \mathbf{T}_\xi \mathbf{U}_{\mathbf{L+R}}^\dagger$ holds, since (6.4) is clearly in the adjoint representation. Using now the bi-doublets defined in (3.30) we can make manifest the $SU(2)_{L+R}$ representations of the Higgs fields, by writing

$$\mathbf{H} + \overline{\mathbf{H}} = \varphi^0 \mathbf{I}_2 + i\sqrt{2} \begin{bmatrix} G^0/\sqrt{2} & G^+ \\ G^- & -G^0/\sqrt{2} \end{bmatrix} \equiv \varphi^0 \mathbf{I}_2 + i\sqrt{2} \mathbf{T}_\mathbf{G}, \quad (6.5)$$

$$-i(\mathbf{H} - \overline{\mathbf{H}}) = a^0 \mathbf{I}_2 - i\sqrt{2} \begin{bmatrix} H^0/\sqrt{2} & H^+ \\ H^- & -H^0/\sqrt{2} \end{bmatrix} \equiv a^0 \mathbf{I}_2 - i\sqrt{2} \mathbf{T}_\mathbf{H}, \quad (6.6)$$

corresponding to the decomposition under $SU(2)_{L+R}$ of a $SU(2)_L \times SU(2)_R$ bi-doublet: $(2, 2) \sim 2 \otimes 2 = 1 \oplus 3$. These formulae show that φ^0 and a^0 are $SU(2)_{L+R}$ singlets [and this is consistent with the fact that there can be $\varphi^0 - t_R^0$ and $a^0 - t_L^0$ mixing; t being obviously a $SU(2)_{L+R}$ singlet too]. Then the three Goldstone bosons form a real $SU(2)_{L+R}$ triplet, $\mathbf{T}_\mathbf{G}$ and have zero masses. Finally, H^\pm and H^0 form another triplet, $\mathbf{T}_\mathbf{H}$. As a consequence, H^0 does not mix with other neutral scalars and is mass-degenerate with H^\pm , up to $\mathcal{O}(g'^2 v^2)$ corrections [see eq. (6.3)].

Note also that, if the v -independent part of $M_{H^0}^2 \simeq M_{H^\pm}^2$ and $M_{A^0}^2$ dominates over the v -dependent part (and this is the case of interest to us, with μ and $m_{3/2}$ larger than v , which is much smaller than Λ), then H^\pm , H^0 and A^0 arrange in a heavy nearly-degenerate $SU(2)_L$ Higgs doublet

$$H = \frac{1}{\sqrt{2}} [H_2 - \overline{H}_1] = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{bmatrix}, \quad (6.7)$$

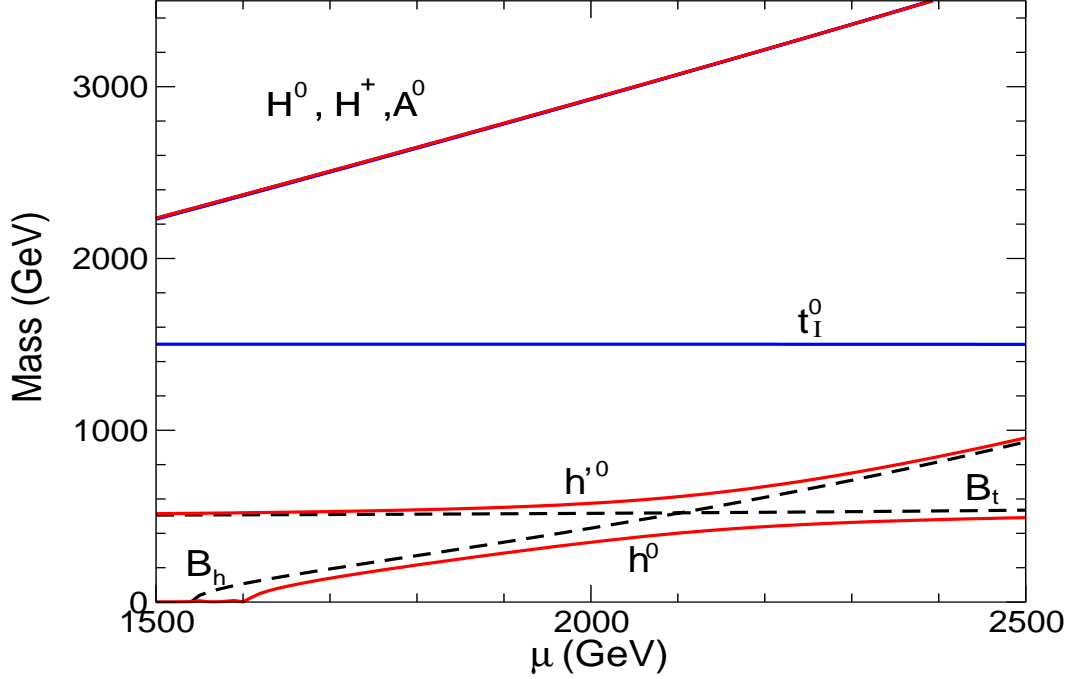


Figure 1: Higgs boson masses, in GeV, as functions of μ for $\Lambda = 4$ TeV, $m_{3/2} = 500$ GeV and $\varsigma = 1 + \sqrt{3}$.

which does not participate in the breaking of the electroweak symmetry¹⁰ and has mass (squared) $M_H^2 \sim 2 [\varsigma \mu m_{3/2} - \lambda(2 + \varsigma)m_{3/2}^2]$. In addition, there is another heavy pseudoscalar, $A'^0 \simeq t_I^0$, with mass $\sim 3m_{3/2}$ and two scalars, φ^0 and t_R^0 , which can mix and be moderately light (the mass eigenstates are h^0, h'^0). This behaviour is clearly shown by the numerical example given in figure 1. We take $\Lambda = 4$ TeV, $m_{3/2} = 500$ GeV, $\varsigma = 1 + \sqrt{3}$ and let μ vary inside the range that gives a proper electroweak breaking. For each value of μ , λ is chosen so as to satisfy $v^2 = (246 \text{ GeV})^2$. In the region shown λ varies from -0.4 to -3 . We present the Higgs masses as a function of μ : H^0, A^0 and H^+ are heavy and appear nearly-degenerate in the upper part of the plot; the other curves give the masses for the two light \mathcal{CP} -even scalars, h^0, h'^0 and the second \mathcal{CP} -odd scalar, which we label t_I^0 because it is basically a pure t_I^0 state. We also show the two upper bounds on M_{h^0} as given by eq. (6.1) (labelled B_h) and eq. (6.2) (labelled B_t).

From figure 1 we see that the lightest Higgs boson, h^0 , can be quite heavy compared

¹⁰The combination relevant for electroweak breaking is the orthogonal one, $\left[\frac{1}{\sqrt{2}}(G^+ + iG^0) \right]$.

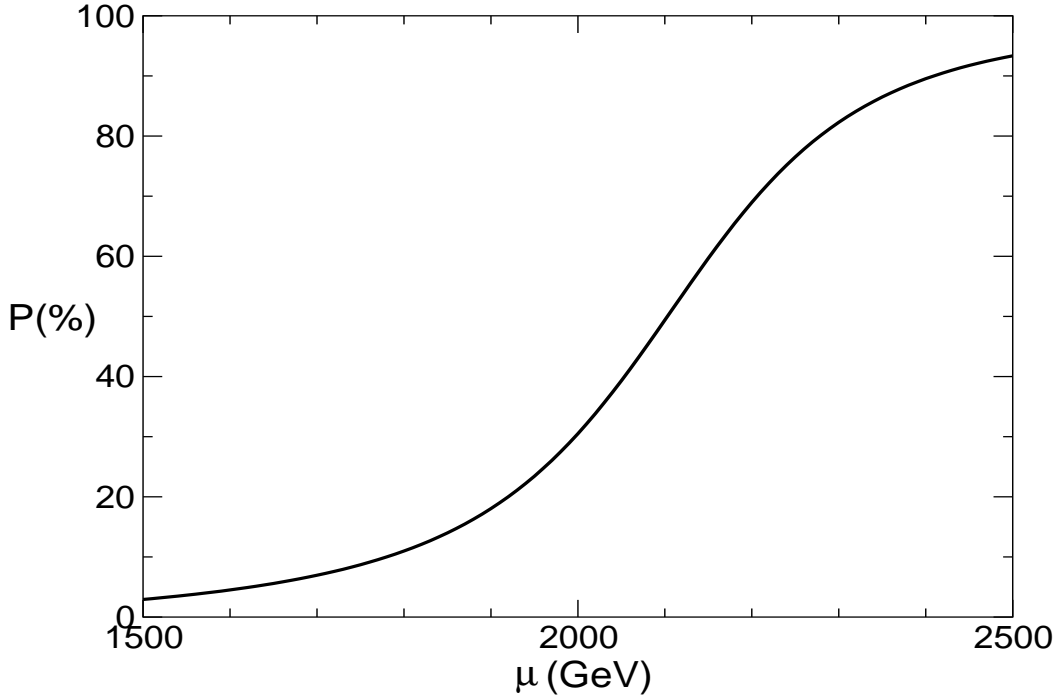


Figure 2: Percentage, P , of singlet (t_R^0) component in the lightest Higgs h^0 . The percentage of singlet component in the second-to-lightest Higgs, h'^0 , is $100\% - P$.

with the MSSM case (up to $M_{h^0} \sim m_{3/2} = 500$ GeV in this particular example). To ascertain the properties of this light Higgs it is necessary to study its composition in terms of φ^0 [which belongs to a $SU(2)_L$ -doublet] and t_R^0 (which is a gauge singlet). This is shown in figure 2, which gives the percentage of singlet component in h^0 (that of h'^0 is exactly complementary to it). In the region of low values of μ , when M_{h^0} approaches B_h , h^0 is mostly φ^0 (*i.e.* SM-like), with a small but non-negligible ($\sim 5\%$) singlet component. For larger values of μ , when M_{h^0} approaches B_t , h^0 is mostly t_R^0 , with a large ($\sim 95\%$) singlet component. In this respect, the true Higgs in this latter region is h'^0 , which is significantly heavier than h^0 . In fact, one should worry about perturbativity if a SM-like Higgs gets as heavy as h'^0 does in that region. If we demand that the quartic coupling $(\lambda_\beta + \xi_\beta)$ remains below ~ 4 (see *e.g.* [31]), we obtain an upper limit $M_{h'^0} \lesssim 700$ GeV, similar to the usual perturbativity limit on the SM-like Higgs boson. (This sets also an upper limit ~ 2250 GeV on the value of μ .) The fact that the Higgs upper bound is so much higher than the usual bound $M_h^0 \lesssim 205$ GeV in general SUSY models [32] is due to the smallness of the cut-off scale Λ (the usual bound assumes the model to be valid all the way up to the Planck scale).

So far we have discussed only tree-level masses. One should add to them radiative corrections, which are expected to be sizeable in general, because the stops have large masses in this model. These top-stop radiative corrections have the same form as in the MSSM [33] and can raise the Higgs mass significantly.

7 Neutralino sector

In this section we present the 5×5 neutralino mass matrix after electroweak symmetry breaking. First there are supersymmetric contributions to Higgsino and radino masses from eq. (3.38). To this one should add gaugino soft masses [M_1 and M_2 for $U(1)_Y$ and $SU(2)_L$, respectively] and the soft-terms of eq. (4.27) which, more explicitly, give the mass terms:

$$\delta\mathcal{L}_{\text{soft}} = \tilde{m}_{3/2} \left[(\chi_t \cdot \chi_t) - \lambda(\chi_{H_1^0} \cdot \chi_{H_2^0}) \right] + \text{h.c.} \quad (7.1)$$

The last step to find the neutralino mass matrix is to normalize the neutralino fields so as to get canonical kinetic terms. This is exactly similar to what was done for the Higgs bosons in Appendix C, and in fact the field-redefinition of the Higgsinos is identical. With all these ingredients, the neutralino mass matrix, in the basis $\{\lambda_B, \lambda_{W_3}, \chi_{H_1^0}, \chi_{H_2^0}, \chi_t\}$ and neglecting $\mathcal{O}(v^2/\Lambda)$ terms, reads:

$$\mathcal{M}_{\tilde{\chi}^0} \simeq \begin{bmatrix} M_1 & 0 & -M_Z s_w c_\beta & M_Z s_w s_\beta & 0 \\ 0 & M_2 & M_Z c_w c_\beta & -M_Z c_w s_\beta & 0 \\ -M_Z s_w c_\beta & M_Z c_w c_\beta & 0 & -\mu & \mu_1 \frac{v}{\Lambda} \\ M_Z s_w s_\beta & -M_Z c_w s_\beta & -\mu & 0 & \mu_2 \frac{v}{\Lambda} \\ 0 & 0 & \mu_1 \frac{v}{\Lambda} & \mu_2 \frac{v}{\Lambda} & 2\tilde{m}_{3/2} \end{bmatrix}, \quad (7.2)$$

where θ_w is the Weinberg angle; we use the short-hand notation $s_\beta = \sin \beta$, $c_w = \cos \theta_w$, etc; and

$$\begin{aligned} \mu_1 &\equiv \frac{1}{4}(1 + \lambda)(2\tilde{m}_{3/2} - \mu) + \sqrt{2}\mu_0 \sin \beta, \\ \mu_2 &\equiv \frac{1}{4}(1 + \lambda)(2\tilde{m}_{3/2} - \mu) + \sqrt{2}\mu_0 \cos \beta. \end{aligned} \quad (7.3)$$

We note that μ_0 has combined with $\lambda\tilde{m}_{3/2}$ to give μ as usual (the same happens in the chargino mass matrix, which is of standard form), and χ_t receives a soft mass $2\tilde{m}_{3/2}$. This solves a possible problem with a very light neutralino, because the SUSY mass

of χ_t is only of order $\mu_0 v^2/\Lambda^2$. Notice also in (7.2) that the gauginos do not have a direct mixing with the radino, the reason being that the latter is a gauge singlet. The structure of the neutralino mass matrix (7.2) is different from that of the NMSSM (see *e.g.* [34] and references therein). Once again the difference can be traced back to the non-minimal Kähler potential.

8 Summary and conclusions

Supersymmetrization of warped scenarios is helpful to avoid some shortcomings of the original Randall-Sundrum construction. In particular, SUSY helps to explain the correlations between the brane tensions and the bulk cosmological constant, and to protect the hierarchy against destabilization by radiative corrections. In addition, SUSY is likely a necessary ingredient to make contact with warped superstring constructions.

The main goal of this paper has been to derive the low-energy effective theories of supersymmetric warped constructions and to extract from them the low-energy phenomenology. They turn out to be pretty unconventional, which might be useful to alleviate some of the drawbacks of the MSSM. An important example is the upper bound on the Higgs mass: while it is starting to be worrisome in the conventional MSSM, we have showed that it completely disappears here. On the other hand, these constructions offer several characteristic phenomenological features, which depart from those of ordinary supersymmetric scenarios.

Next we summarize the main results and conclusions of the paper.

Effective supersymmetric theory

As discussed in sect.2, the most distinctive feature of warped supersymmetric constructions with respect to ordinary supersymmetric versions of the SM is the fairly model-independent fashion in which the radion couples to the visible fields, *both* in the superpotential and the Kähler potential. These are given by eq.(2.11) [or alternatively eq.(2.13)] and eq.(2.10). The first equation still leaves room for some arbitrariness in the form of the Kähler potential, K . For practical computations we have used the form of K inspired by ref. [6], which corresponds to eq.(2.14). For other ansätze, most of the basic results hold, as they are a consequence of the above-mentioned model-independent features. This is briefly illustrated in Appendix B for an alternative, and

somewhat simpler, choice of K .

In our scheme, all the visible fields, including the ordinary Yang-Mills fields, live in the visible brane. We have included, however, a universal hypermultiplet in the bulk, which plays the role of the usual moduli fields (in particular the dilaton), abundant in superstring constructions. As discussed in sect.3, we have not made any assumption about the form of the gauge kinetic functions, f_A . They are not very important for the phenomenology discussed here, except for the predictions on gaugino masses.

The most important novelty of the effective supersymmetric theory is that the Kähler potential is not minimal in the $M_p \rightarrow \infty$ limit. This is illustrated by eq.(3.3) in our particular scenario. As a result the low-energy globally-supersymmetric theory (whose general Lagrangian is given in Appendix A) shows distinctive features. In particular, the radion mixes with the Higgs fields (and the radino with the Higgsinos) through kinetic and mass terms after electroweak symmetry breaking. Also, there arise new supersymmetric couplings (or corrections to ordinary couplings), suppressed only by μ_0/Λ (or $m_{3/2}/\Lambda$ after SUSY breaking), where $\Lambda = \mathcal{O}(\text{TeV})$ is the typical mass scale in the visible brane, given by eq.(3.1), and μ_0 is the μ -parameter in the superpotential. [Incidentally, there is no μ -problem in these scenarios since, besides the known mechanisms to generate it, its natural value is $\mathcal{O}(\Lambda)$, as any other mass scale in the visible brane.] These new couplings include extra quartic couplings for the Higgses, which completely change the Higgs phenomenology (a complete analysis, however, requires to take into account the SUSY breaking contributions, see below). Finally, there are higher order operators (*e.g.* with two derivatives) suppressed only by inverse powers of Λ . These give rise to tree-level contributions to the ρ parameter, setting lower bounds on the value of Λ , see eq.(3.29).

Finally, let us note that in the $\Lambda \rightarrow \infty$ limit, the radion decouples and the superpotential and the Kähler potential recover their ordinary “minimal” forms.

Supersymmetry breaking

SUSY breaking plays an essential role for the stabilization of the radion field and is mandatory for a correct low-energy phenomenology. In our scheme SUSY is broken by brane superpotentials $W_{sbr} = W_h + e^{-3T}W_v$ located at the two branes. This is similar to the scheme of ref. [6], but with one important difference: we allow the superpotentials to depend on the “dilaton” field, S (i.e. the bulk moduli). Then SUSY *can* be broken

at vanishing cosmological constant (this constrains the form of the S -dependence). The T field gets stabilized with a VEV related to the SUSY breaking scale. Provided the dependence of W_{sbr} on S is factorizable, $F_T = 0$ and the breaking occurs entirely along the F_S direction.

The corresponding soft terms show universality as a result of the flavour blindness of the S -couplings. They are more or less conventional, except for the appearance of non-standard trilinear terms of the form $t^*|\varphi|^2 + \text{h.c.}$ (where φ is a chiral scalar field) and dimension 4 (and higher) SUSY-breaking operators suppressed only by powers of $m_{3/2}/\Lambda$. We show that they do not introduce dangerous quadratic divergences able to spoil the naturalness of the electroweak breaking. However, they play a relevant role in the phenomenology of these scenarios.

Electroweak breaking

Due to the universality of the soft breaking terms (in particular the Higgs mass terms) and the short range for the RG running, $\tan\beta \simeq 1$ appears as a typical feature of these models. In an ordinary MSSM this would be at odds with the present bounds on the lightest Higgs boson. Here this is not so, since electroweak breaking takes place in a rather unconventional way. It remains a fact that proper electroweak breaking requires that the origin of the Higgs-field space is destabilized, which implies $m_1^2 m_2^2 - m_{12}^4 < 0$. However, the usual complementary condition $m_1^2 + m_2^2 + 2m_{12}^2 > 0$, to avoid an unbounded from below (D -flat) direction along $H_1^0 = H_2^0$, is not required anymore. This is because, as stated above, $V(H)$ contains extra quartic couplings which lift the D -flat direction. A complete expression for the Higgs potential is given in eqs.(5.1–5.3).

These facts imply that electroweak breaking occurs at tree-level (it is not a radiative effect, like in the MSSM). Still, the breaking is completely natural in the sense that the Higgs fields are the only ones with a μ -term (and the corresponding bilinear soft term), and therefore (provided the soft breaking masses are positive) are the only scalar fields of the theory which can be destabilized at the origin.

Higgs and neutralino sectors

Due to the unconventional electroweak breaking the usual expressions for the Higgs spectrum do not hold any longer. This is true, in particular, for the usual MSSM tree-level bound on the mass of the lightest Higgs field, $M_{h^0}^2 \leq M_Z^2 \cos^2 2\beta$, which is

now replaced by the bounds of eqs.(6.1, 6.2). Actually, we show that M_{h^0} can be as large as 700 GeV, a size similar to the usual perturbativity limit on the SM-like Higgs boson.

The complete spectrum of the Higgs sector is a bit more complicated than in the MSSM, since two additional real fields (one \mathcal{CP} -even and one \mathcal{CP} -odd) enter the game, namely the real and imaginary parts of the radion field. (The complete expressions for the tree-level spectrum are given in Appendix C.) Actually, another important difference with ordinary cases is that the neutral Higgs field eigenstates get a sizeable component of the radion. In particular, the percentage of radion in the lightest Higgs field, h^0 , depends on the value of μ , but it is non-negligible in all cases. This affects the experimental properties of the light Higgs field, since the radion is a singlet.

Likewise, as stated above, the Higgsinos get mixed with the radino. After the SUSY and electroweak breakdown, this results in a non-conventional 5×5 neutralino mass matrix, which is explicitly given in eq.(7.2).

To conclude, the low-energy effective theory from supersymmetric warped constructions presents many interesting differences (which are sometimes advantages) from the ordinary MSSM. They are caused by the non-minimal (and quite model-independent) form of the superpotential and the Kähler potential, a fact that could be also present in other models with extra-dimensions where the fundamental scale is $\mathcal{O}(\text{TeV})$. This offers novel ways of accommodating and testing new physics.

Acknowledgments

We thank A. Falkowski, S. Lalak, C. Muñoz and R. Sundrum for very useful discussions and correspondence.

A. Lagrangian for general Kähler potentials

Given a superpotential $W(\Phi)$ and a general Kähler potential $K(\Phi, \Phi^\dagger)$ the corresponding supersymmetric Lagrangian is:

$$\mathcal{L} = \frac{1}{2} \left\{ 2 [W(\Phi)]_{\mathcal{F}} + \frac{1}{2} [K[\Phi, \Phi^\dagger \exp(-2\mathbf{t}_A V_A)]_D - \frac{1}{2} [f_{AB}(\Phi)(W_{AL}^T \epsilon W_{BL})]_{\mathcal{F}} \right\} + h.c. \quad (\text{A.1})$$

Here, \mathbf{t}_A are Hermitian generators of the gauge group algebra, labelled by A ; V_A are the corresponding gauge vector superfields; W_{AL} are the chiral field-strength spinor superfields; $\epsilon_{\alpha\beta}$ is the totally antisymmetric tensor and $f_{AB}(\Phi)$ is the gauge kinetic function.

Assuming further that W and K do not depend on field derivatives, the Lagrangian (A.1) written in components reads ($\mathcal{G}_{ij} \equiv \partial^2 K / \partial \phi_i \partial \phi_j^*$):

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left\{ \mathcal{G}_{ij} [i\zeta_j^\dagger \boldsymbol{\sigma}^\mu D_\mu \zeta_i + \mathcal{F}_i \mathcal{F}_j^* + D_\mu \phi_i D^\mu \phi_j^*] - i \frac{\partial^3 K(\phi, \phi^*)}{\partial \phi_i \partial \phi_j \partial \phi_l^*} (\zeta_i^T \boldsymbol{\sigma}_2 \zeta_j) \mathcal{F}_l^* \right. \\ & + i \frac{\partial^3 K(\phi, \phi^*)}{\partial \phi_i \partial \phi_j \partial \phi_l^*} (\zeta_l^\dagger \boldsymbol{\sigma}^\mu \zeta_j) D_\mu \phi_i + \frac{1}{4} \frac{\partial^4 K(\phi, \phi^*)}{\partial \phi_i \partial \phi_j \partial \phi_l^* \partial \phi_k^*} (\zeta_i^T \boldsymbol{\sigma}_2 \zeta_j) (\zeta_k^\dagger \boldsymbol{\sigma}_2 \zeta_l^*) \\ & - i \frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j} (\zeta_i^T \boldsymbol{\sigma}_2 \zeta_j) + 2\mathcal{F}_i \frac{\partial W(\phi)}{\partial \phi_i} - \frac{1}{4} (\lambda_A^T \boldsymbol{\sigma}_2 \lambda_B) (\zeta_i^T \boldsymbol{\sigma}_2 \zeta_j) \frac{\partial^2 f_{AB}(\phi)}{\partial \phi_i \partial \phi_j} \\ & - \frac{i}{2} (\lambda_A^T \boldsymbol{\sigma}_2 \lambda_B) \mathcal{F}_i \frac{\partial f_{AB}(\phi)}{\partial \phi_i} + \frac{\sqrt{2}}{4} \frac{\partial f_{AB}(\phi)}{\partial \phi_i} [i(\lambda_B^T \boldsymbol{\sigma}_2 \bar{\boldsymbol{\sigma}}^\mu \boldsymbol{\sigma}^\nu \zeta_i) f_{A\mu\nu} - 2(\lambda_B^T \boldsymbol{\sigma}_2 \zeta_i) D_A] \\ & + f_{AB}(\phi) \left[-i\lambda_B^\dagger \boldsymbol{\sigma}^\mu D_\mu \lambda_A - \frac{1}{4} F_{A\mu\nu} F_B^{\mu\nu} + \frac{i}{8} \epsilon_{\mu\nu\rho\sigma} F_A^{\mu\nu} F_B^{\rho\sigma} + \frac{1}{2} D_A D_B \right] \\ & \left. - \frac{\partial K(\phi, \phi^*)}{\partial \phi_i^*} D_A (\phi^\dagger \mathbf{t}_A)_i + 2\sqrt{2} \frac{\partial^2 K(\phi, \phi^*)}{\partial \phi_i \partial \phi_j^*} (\mathbf{t}_A \phi)_i (\zeta_j^\dagger \boldsymbol{\sigma}_2 \lambda_A^*) \right\} + h.c. \quad (\text{A.2}) \end{aligned}$$

with $\Phi_i = (\phi_i, \zeta_i, \mathcal{F}_i)$, $V_A = (V_{A\mu}, \lambda_A, D_A)$; $D_\mu \phi_i \equiv \partial_\mu \phi_i - i(\mathbf{t}_A \phi)_i V_\mu^A$ and a similar definition for $D_\mu \zeta_i$. Eq. (A.2) corrects formula (27.4.42) of [20] by including the gauge terms of $1/2[K]_D$ which are not absorbed in the replacement $\partial_\mu \rightarrow D_\mu$, *i.e.* the term with a single derivative of K and the K -dependent term that mixes matter fermions with gauginos [they are collected in the last line of (A.2)].

The auxiliary fields \mathcal{F}_i and D_A can be eliminated from \mathcal{L} above by using their equations of motion, which give:

$$\begin{aligned} \mathcal{F}_i^* &= \mathcal{G}_{ij}^{-1} \left\{ -\frac{\partial W(\phi)}{\partial \phi_j} + \frac{i}{2} \frac{\partial^3 K(\phi, \phi^*)}{\partial \phi_k^* \partial \phi_l^* \partial \phi_j} (\zeta_l^\dagger \boldsymbol{\sigma}_2 \zeta_k^*) + \frac{i}{2} \frac{\partial f_{AB}(\phi)}{\partial \phi_j} (\lambda_A^T \boldsymbol{\sigma}_2 \lambda_B) \right\}, \\ D_A &= f_{AB}^{-1}(\phi) \left[\frac{\partial K(\phi, \phi^*)}{\partial \phi_i} (\mathbf{t}_B \phi)_i + \frac{1}{\sqrt{2}} \frac{\partial f_{BC}(\phi)}{\partial \phi_i} (\lambda_C^T \boldsymbol{\sigma}_2 \zeta_i) \right]. \quad (\text{A.3}) \end{aligned}$$

B. Alternative choice of Kähler potential

Throughout the paper we have used the choice (2.14) for the general expression of K , (2.13), which corresponds to the ansatz given in ref. [6]. In the $M_p \rightarrow \infty$ limit this goes to (3.3). In this Appendix we take $\Phi_{\text{vis}} = \sum_i |\hat{\varphi}_i|^2 + (\lambda \hat{H}_1 \cdot \hat{H}_2 + \text{h.c.})$, to be plugged in the general expression of K , (2.11). This is perhaps closer in spirit to the derivation of K given in [4]. Then, in the $M_p \rightarrow \infty$ limit we obtain

$$K_{\text{eff}} = \Lambda^2 \exp \left[-\frac{t+t^*}{\Lambda} \right] \left\{ 1 + \frac{1}{\Lambda^2} \left[\sum_i |\varphi_i|^2 + (\lambda H_1 \cdot H_2 + \text{h.c.}) \right] \right\} \quad (\text{B.1})$$

We can use the formulae given in the text for generic Σ_{eff} if we make the replacement

$$\Sigma_{\text{eff}} \rightarrow \Lambda^2 \log[1 + \Sigma_{\text{eff}}/\Lambda^2] , \quad (\text{B.2})$$

which transforms (3.3) into (B.1). The supersymmetric scalar potential is obtained directly from (3.13) by making such replacement and working out the inverse matrix. The final result is

$$\begin{aligned} V_{\text{SUSY}} = & e^{(t+t^*)/\Lambda} \sum_i \left| \frac{\partial W_{\text{eff}}}{\partial \varphi_i} \right|^2 + \frac{1}{K_{\text{eff}} N_{\text{eff}}} \left| \frac{\partial W_{\text{eff}}}{\partial \phi_i} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i^*} - 3W_{\text{eff}} \right|^2 \\ & + \frac{1}{4} e^{-2(t+t^*)/\Lambda} \left\{ f_A^{-1}(\phi) \left[\frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} (\mathbf{t}_{\mathbf{A}} \varphi)_i \right]^2 + \text{h.c.} \right\} . \end{aligned} \quad (\text{B.3})$$

In this equation, Σ_{eff} is as given in (3.4) and we have defined

$$N_{\text{eff}} \equiv 1 + \frac{\Sigma_{\text{eff}}}{\Lambda^2} - \frac{1}{\Lambda^2} \sum_i \left| \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i} \right|^2 = 1 - \frac{1}{\Lambda^2} \left\{ |\lambda|^2 (|H_1|^2 + |H_2|^2) + (\lambda H_1 \cdot H_2 + \text{h.c.}) \right\} . \quad (\text{B.4})$$

With the same superpotential (3.5), we can obtain from (B.3) the supersymmetric effective potential for H_i and t , which reads:

$$\begin{aligned} V(H_i, t) = & e^{-2(t+t^*)/\Lambda} \left\{ \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} (|H_1|^2 |H_2|^2 - |H_1 \cdot H_2|^2)^2 \right. \\ & \left. + |\mu_0|^2 \frac{(|H_1|^2 + |H_2|^2) [1 - 2(\lambda H_1 \cdot H_2 + \text{h.c.})/\Lambda^2] + |H_1 \cdot H_2|^2/\Lambda^2}{1 - |\lambda|^2 (|H_1|^2 + |H_2|^2)/\Lambda^2 - (\lambda H_1 \cdot H_2 + \text{h.c.})/\Lambda^2} \right\} . \end{aligned} \quad (\text{B.5})$$

This would replace (3.15). The H_i -dependent part of this potential, expanded in powers of H_i/Λ and keeping only renormalizable terms reads:

$$\begin{aligned} V(H_i) = & |\mu_0|^2 (|H_1|^2 + |H_2|^2) + \frac{|\mu_0|^2}{\Lambda^2} |H_1 \cdot H_2 - \lambda^* (|H_1|^2 + |H_2|^2)|^2 \\ & + \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g^2 (|H_1|^2 |H_2|^2 - |H_1 \cdot H_2|^2)^2 , \end{aligned} \quad (\text{B.6})$$

which would replace (3.17).

The scalar effective potential after inclusion of supersymmetry-breaking terms is straightforward to obtain. There is a part quadratic in $m_{3/2}$:

$$\delta V_{\text{soft}} = m_{3/2}^2 \left\{ \frac{\Lambda^2}{N_{\text{eff}}} e^{-2(t+t^*)/\Lambda} - \Lambda^2 \left[e^{-3t/\Lambda} + e^{-3t^*/\Lambda} \right] + K_{\text{eff}} \right\} . \quad (\text{B.7})$$

and a part linear in $\tilde{m}_{3/2}$:

$$\delta V_{\text{soft}} = \tilde{m}_{3/2} \left\{ \frac{e^{(t+t^*)/\Lambda}}{N_{\text{eff}}} \left[\frac{\partial W_{\text{eff}}}{\partial \varphi_i} \frac{\partial \Sigma_{\text{eff}}}{\partial \varphi_i^*} - 3W_{\text{eff}}^* \right] e^{-3t/\Lambda} + W_{\text{eff}}^* (1 - \varsigma) \right\} + \text{h.c.} \quad (\text{B.8})$$

These equations replace (4.21) and (4.22). The SUSY-breaking contribution to the $H_i - t$ effective potential is long but straightforward to obtain from (B.7) and (B.8) and we do not give it explicitly.

Regarding electroweak symmetry breaking, the situation is similar to the one presented in section 5. From the minimization conditions and assuming that all the parameters are real, we obtain again a first condition for v as

$$v^2 = \frac{-m_\beta^2}{\lambda_\beta} , \quad (\text{B.9})$$

with m_β^2 as given in (5.8) but now

$$\lambda_\beta \equiv \frac{1}{8}(g^2 + g'^2) \cos^2 2\beta + \frac{1}{4\Lambda^2} [-2\lambda\mu + (\mu - 3\lambda m_{3/2}) \sin 2\beta]^2 , \quad (\text{B.10})$$

and, a second condition for $\tan \beta$:

$$\begin{aligned} 0 = & \cos 2\beta \left\{ 4m_{3/2}\Lambda^2 \left[\lambda(2 + \varsigma)m_{3/2} - \varsigma\mu \right] \right. \\ & \left. + v^2(3\lambda m_{3/2} - \mu) \left[2\lambda\mu + (2\lambda m_{3/2} - \mu) \sin 2\beta \right] \right\} , \end{aligned} \quad (\text{B.11})$$

where, as before, $\mu \equiv \mu_0 + \lambda m_{3/2}$. In addition we must impose $m_\beta^2 < 0$ and $\lambda_\beta > 0$. The latter is easily satisfied now, because $\lambda_\beta \geq 0$, as can be seen from (B.10); and the former, $m_\beta^2 < 0$, taken together with (B.11), leads to

$$\tan \beta = 1. \quad (\text{B.12})$$

The $\tan \beta \neq 1$ solution of eq.(B.11) is not acceptable for the same reasons we found on section 5.

For the Lagrangian of the fermionic sector, we use the general formula (3.37) for the SUSY part and (4.27) for the SUSY-breaking piece. After the replacement (B.2), we obtain

$$\begin{aligned}\delta\mathcal{L}_{\text{SUSY}} &= \frac{1}{2} \left\{ -\frac{\partial^2 W_{\text{eff}}}{\partial\varphi_i\partial\varphi_j} + \frac{1}{N_{\text{eff}}} \left[3W_{\text{eff}} - \frac{\partial W_{\text{eff}}}{\partial\varphi_k} \frac{\partial\Sigma_{\text{eff}}}{\partial\varphi_k^*} \right] \frac{1}{\Lambda^2} \frac{\partial^2\Sigma_{\text{eff}}}{\partial\varphi_i\partial\varphi_j} \right\} (\chi_i \cdot \chi_j) \\ &+ \frac{2}{\Lambda} \frac{\partial W_{\text{eff}}}{\partial\varphi_i} (\chi_i \cdot \chi_t) - \frac{3W_{\text{eff}}}{\Lambda^2} (\chi_t \cdot \chi_t) + \text{h.c.}\end{aligned}\quad (\text{B.13})$$

and

$$\delta\mathcal{L}_{\text{soft}} = \frac{1}{2} \tilde{m}_{3/2} e^{-3t/\Lambda} \left\{ -\frac{1}{N_{\text{eff}}} \frac{\partial^2\Sigma_{\text{eff}}}{\partial\varphi_i\partial\varphi_j} (\chi_i \cdot \chi_j) + 2(\chi_t \cdot \chi_t) \right\} + \text{h.c.} \quad (\text{B.14})$$

To simplify these (and previous) expressions we have made use of $[\partial^2\Sigma_{\text{eff}}/\partial\varphi_i\partial\varphi_j^*]^{-1} = \delta_{ij}$ and $[\partial^3\Sigma_{\text{eff}}/\partial\varphi_i\partial\varphi_j\partial\varphi_l^*]^{-1} = 0$. The conclusions for the neutralino sector are qualitatively similar to those of the case discussed in the main text

C. Higgs spectrum

The 3×3 mass matrix $\mathbf{M}_{\mathbf{h}}^2$ for \mathcal{CP} -even Higgses, in the basis $\{\phi_i\} = \{h_1^{0r}, h_2^{0r}, t_R^0\}$, with $h_{1,2}^{0r} \equiv \sqrt{2}\text{Re}[H_{1,2}^0]$ and $t_R^0 \equiv \sqrt{2}\text{Re}[t]$ is

$$[\mathbf{M}_{\mathbf{h}}^2]_{ij}^{(0)} \equiv \left[\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} - \delta_{ij} \frac{1}{\langle \phi_i \rangle} \frac{\partial V}{\partial \phi_i} \right] \Big|_{\phi_j = \langle \phi_j \rangle}, \quad (\text{C.1})$$

where we have subtracted the (zero) tadpoles to get a simpler expression. Instead of giving the matrix elements in the basis $\{\phi_i\}$, it proves useful to change to the following rotated basis $\{\varphi^0 \equiv (h_1^{0r} + h_2^{0r})/\sqrt{2}, H^0 \equiv (h_1^{0r} - h_2^{0r})/\sqrt{2}, t_R^0\}$, in which the matrix elements are:

$$\begin{aligned} \langle H^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | H^0 \rangle &= 2 \left[\varsigma \mu m_{3/2} - \lambda(2 + \varsigma) m_{3/2}^2 \right] + \frac{1}{4} (g^2 + g'^2) v^2 \\ &- v^2 \frac{m_{3/2}^2}{\Lambda^2} \left[\left(\lambda + \frac{13}{2} \lambda^2 \right) - (2 + 7\lambda - 3\lambda^2) \frac{\mu}{m_{3/2}} - \left(4\lambda + \frac{3}{2} \right) \frac{\mu^2}{m_{3/2}^2} \right], \end{aligned} \quad (\text{C.2})$$

$$\langle H^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | \varphi^0 \rangle = 0, \quad (\text{C.3})$$

$$\langle H^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle = 0, \quad (\text{C.4})$$

$$\begin{aligned} \langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | \varphi^0 \rangle &= 2\lambda_\beta v^2 = v^2 \frac{m_{3/2}^2}{\Lambda^2} \left[\left(\frac{13}{2} \lambda^2 + 2\lambda \right) \right. \\ &+ \left. \left(6\lambda^2 - \frac{7}{2} \lambda - 4 \right) \frac{\mu}{m_{3/2}} - \left(8\lambda + \frac{7}{2} \right) \frac{\mu^2}{m_{3/2}^2} \right], \end{aligned} \quad (\text{C.5})$$

$$\langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle = \frac{v}{\sqrt{2}} \frac{m_{3/2}^2}{\Lambda} \left[-2 - (7 + 3\varsigma)\lambda + (1 + 3\varsigma) \frac{\mu}{m_{3/2}} - 4 \frac{\mu^2}{m_{3/2}^2} \right], \quad (\text{C.6})$$

$$\langle t_R^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle = m_{3/2}^2 + v^2 \frac{m_{3/2}^2}{\Lambda^2} \left[1 + \frac{9}{4} \lambda(3 + \varsigma) - \frac{1}{4} (7 + 9\varsigma) \frac{\mu}{m_{3/2}} + 4 \frac{\mu^2}{m_{3/2}^2} \right]. \quad (\text{C.7})$$

The rest of elements not shown follow from the symmetric nature of $\mathbf{M}_{\mathbf{h}(0)}^2$. By using this rotated matrix we have exposed H^0 as an exact eigenstate of $\mathbf{M}_{\mathbf{h}(0)}^2$, with eigenvalue (C.2). The other two eigenvalues are simply extracted from the 2×2 submatrix for $\varphi^0 - t_R^0$.

The eigenvalues of $\mathbf{M}_{\mathbf{h}(0)}^2$ are not yet the true tree-level Higgs masses because a non-minimal Kähler potential leads, after symmetry breaking, to non-canonical kinetic terms for scalar fields (see the discussion in subsection 3.2). More precisely, eq. (3.23)

gives, for the neutral Higgses

$$\begin{aligned}
\delta\mathcal{L}_{\text{kin}} &= \left(\partial_\mu H_1^0 \partial^\mu H_1^{0*} + \partial_\mu H_2^0 \partial^\mu H_2^{0*} \right) \left[1 + (\lambda + 1)(\lambda + 3) \frac{v^2}{4\Lambda^2} \right] \\
&+ \partial_\mu t \partial^\mu t^* \left[1 + (\lambda + 1) \frac{v^2}{2\Lambda^2} \right] - \frac{(1 + \lambda)v}{2\Lambda} \left[\partial_\mu t^* \left(\partial^\mu H_1^0 + \partial^\mu H_2^0 \right) + \text{h.c.} \right] \\
&+ (1 + \lambda)^2 \frac{v^2}{4\Lambda^2} \left[\partial_\mu H_1^0 \partial^\mu H_2^{0*} + \text{h.c.} \right] + \mathcal{O} \left(\frac{v^3}{\Lambda^3} \right) .
\end{aligned} \tag{C.8}$$

Notice that, for the particular value $\lambda = -1$ all effects associated with non-canonical kinetic terms disappear. The kinetic terms in (C.8) are recast in canonical form by the field redefinition:

$$\begin{bmatrix} H_1^0 \\ H_2^0 \\ t \end{bmatrix} \rightarrow \mathbf{N} \begin{bmatrix} H_1^0 \\ H_2^0 \\ t \end{bmatrix} , \tag{C.9}$$

with \mathbf{N} a non-singular matrix with positive eigenvalues of the form $\mathbf{N} = \mathbf{R}^{-1} \mathbf{S} \mathbf{R}$, where \mathbf{R} is a field rotation that makes diagonal the kinetic terms, \mathbf{S} is a field re-scaling to get canonical coefficients in the diagonalized kinetic terms, and \mathbf{R}^{-1} rotates the fields back so as to obtain $\mathbf{N} = \mathbf{I}_3$ if the kinetic mixing terms in (C.8) were switched off. The explicit form of \mathbf{N} is

$$\begin{aligned}
\mathbf{N} &= \mathbf{I}_3 + \frac{v}{4\Lambda} (1 + \lambda) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \\
&- \frac{v^2}{32\Lambda^2} (1 + \lambda) \begin{bmatrix} (9 + \lambda) & (1 + \lambda) & 0 \\ (1 + \lambda) & (9 + \lambda) & 0 \\ 0 & 0 & 2(1 - 3\lambda) \end{bmatrix} + \mathcal{O} \left(\frac{v^3}{\Lambda^3} \right) .
\end{aligned} \tag{C.10}$$

This transformation changes the Higgs mass matrices. In particular, the new mass matrix for \mathcal{CP} -even Higgses is

$$\mathbf{M}_{\text{h}}^2 = \mathbf{N}^T \mathbf{M}_{\text{h}(0)}^2 \mathbf{N} . \tag{C.11}$$

The elements of this new matrix have a simple form in terms of those of the initial matrix $\mathbf{M}_{\text{h}(0)}^2$ [eqs. (C.2)-(C.7)]. Note that we are discussing here an effect up to order v^2/Λ^2 , which in principle is small, but it is necessary to take it into account if we want to know the Higgs masses to order $m^2 v^2/\Lambda^2$ (with $m^2 = m_{3/2}^2$, $\mu m_{3/2}$, or μ^2). Neglecting contributions of order $m^2 v^4/\Lambda^4$ we get

$$\langle H^0 | \mathbf{M}_{\text{h}}^2 | H^0 \rangle = \langle H^0 | \mathbf{M}_{\text{h}(0)}^2 | H^0 \rangle \left[1 - \frac{v^2}{2\Lambda^2} (1 + \lambda) \right] + \dots \equiv M_{H^0}^2$$

$$\begin{aligned}
&= 2 \left[\varsigma \mu m_{3/2} - \lambda(2 + \varsigma) m_{3/2}^2 \right] + \frac{1}{4} (g^2 + g'^2) v^2 \\
&+ v^2 \frac{m_{3/2}^2}{\Lambda^2} \left\{ \lambda \left[1 - \frac{9}{2} \lambda + \varsigma(1 + \lambda) \right] \right. \\
&- \left. \left[\varsigma - 2 + (\varsigma - 7) \lambda + 3 \lambda^2 \right] \frac{\mu}{m_{3/2}} + \frac{1}{2} (3 + 8 \lambda) \frac{\mu^2}{m_{3/2}^2} \right\}, \quad (C.12)
\end{aligned}$$

$$\langle H^0 | \mathbf{M}_{\mathbf{h}}^2 | \varphi^0 \rangle = 0, \quad (C.13)$$

$$\langle H^0 | \mathbf{M}_{\mathbf{h}}^2 | t_R^0 \rangle = 0, \quad (C.14)$$

$$\begin{aligned}
\langle \varphi^0 | \mathbf{M}_{\mathbf{h}}^2 | \varphi^0 \rangle &\equiv 2(\lambda_\beta + \xi_\beta) v^2 = \langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | \varphi^0 \rangle \\
&+ \frac{v}{\sqrt{2} \Lambda} (1 + \lambda) \langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \frac{v^2}{8 \Lambda^2} (1 + \lambda)^2 \langle t_R^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \dots \\
&= v^2 \frac{m_{3/2}^2}{2 \Lambda^2} \left[-\frac{7}{4} - \frac{3}{2} (3 + 2 \varsigma) \lambda + \left(\frac{25}{4} - 3 \varsigma \right) \lambda^2 \right. \\
&- \left. \left[7 + 13 \lambda - 12 \lambda^2 - 3 \varsigma (1 + \lambda) \right] \frac{\mu}{m_{3/2}} - (11 + 20 \lambda) \frac{\mu^2}{m_{3/2}^2} \right], \quad (C.15)
\end{aligned}$$

$$\begin{aligned}
\langle \varphi^0 | \mathbf{M}_{\mathbf{h}}^2 | t_R^0 \rangle &= \langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \frac{v}{2 \sqrt{2} \Lambda} (1 + \lambda) \langle t_R^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \dots \\
&= -\frac{v}{\sqrt{2}} \frac{m_{3/2}^2}{\Lambda} \left[\frac{3}{2} + \left(\frac{13}{2} + 3 \varsigma \right) \lambda - (1 + 3 \varsigma) \frac{\mu}{m_{3/2}} + 4 \frac{\mu^2}{m_{3/2}^2} \right], \quad (C.16)
\end{aligned}$$

$$\begin{aligned}
\langle t_R^0 | \mathbf{M}_{\mathbf{h}}^2 | t_R^0 \rangle &= \langle t_R^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \frac{v}{\sqrt{2} \Lambda} (1 + \lambda) \langle \varphi^0 | \mathbf{M}_{\mathbf{h}(0)}^2 | t_R^0 \rangle + \dots \\
&= m_{3/2}^2 + v^2 \frac{m_{3/2}^2}{8 \Lambda^2} \left[-1 + (20 + 6 \varsigma) \lambda - (25 + 12 \varsigma) \lambda^2 \right. \\
&+ \left. 2[-5 + 2 \lambda + 3(2 \lambda - 1) \varsigma] \frac{\mu}{m_{3/2}} + 16(1 - \lambda) \frac{\mu^2}{m_{3/2}^2} \right]. \quad (C.17)
\end{aligned}$$

We find that H^0 is still an eigenstate of the new mass matrix $\mathbf{M}_{\mathbf{h}}^2$ with an eigenvalue, M_{H^0} , only slightly different, while the $\varphi^0 - t_R^0$ submatrix is somewhat changed.

The 3×3 mass matrix $\mathbf{M}_{\mathbf{A}(0)}^2$ for \mathcal{CP} -odd Higgses, is obtained by the same formulae (C.1) and (C.11), with $\{\phi_i\} = \{h_1^{0i}, h_2^{0i}, t_I^0\}$ and $h_{1,2}^{0i} \equiv \sqrt{2} \text{Im}[H_{1,2}^0]$, $t_I^0 \equiv \sqrt{2} \text{Im}[t]$. In this case it is also useful to perform a change of basis and work with $\{G^0 \equiv (h_1^{0i} - h_2^{0i})/\sqrt{2}, a^0 \equiv (h_1^{0i} + h_2^{0i})/\sqrt{2}, t_I^0\}$. The neutral Goldstone boson, G^0 , is exposed as an eigenstate of this matrix, with zero eigenvalue. The remaining 2×2 submatrix has elements

$$\langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | a^0 \rangle = 2 \left[\varsigma \mu m_{3/2} - \lambda(2 + \varsigma) m_{3/2}^2 \right]$$

$$- v^2 \frac{m_{3/2}^2}{\Lambda^2} \left[\lambda(1+8\lambda) - (2+10\lambda-3\lambda^2) \frac{\mu}{m_{3/2}} - 4\lambda \frac{\mu^2}{m_{3/2}^2} \right], \quad (\text{C.18})$$

$$\langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle = 3 \frac{v}{\sqrt{2}} \frac{m_{3/2}^2}{\Lambda} (\varsigma - 1) \left(\lambda - \frac{\mu}{m_{3/2}} \right), \quad (\text{C.19})$$

$$\langle t_I^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle = 9m_{3/2}^2 + 9v^2 \frac{m_{3/2}^2}{\Lambda^2} (1 - \varsigma) \left(\lambda - \frac{\mu}{m_{3/2}} \right). \quad (\text{C.20})$$

The field redefinition (C.9) affects also the mass matrix for pseudoscalars. The elements of the re-scaled matrix $\mathbf{M}_{\mathbf{A}}^2 \equiv \mathbf{N}^T \mathbf{M}_{\mathbf{A}(0)}^2 \mathbf{N}$, neglecting again terms of order $m^2 v^4 / \Lambda^4$, are¹¹

$$\begin{aligned} \langle a^0 | \mathbf{M}_{\mathbf{A}}^2 | a^0 \rangle &= \langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | a^0 \rangle \left[1 - \frac{v^2}{4\Lambda^2} (1+\lambda)(5+\lambda) \right] \\ &+ \frac{v}{\sqrt{2}\Lambda} (1+\lambda) \langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle + \frac{v^2}{8\Lambda^2} (1+\lambda)^2 \langle t_I^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle + \dots \\ &= 2 \left[\varsigma \mu m_{3/2} - \lambda(2+\varsigma) m_{3/2}^2 \right] \\ &+ v^2 \frac{m_{3/2}^2}{8\Lambda^2} \left\{ 9 + (18+22\varsigma)\lambda + (24\varsigma-43)\lambda^2 + 2(2+\varsigma)\lambda^3 \right. \\ &\left. - 2 \left[(11+12\lambda+\lambda^2)\varsigma - 2(7+23\lambda-6\lambda^2) \right] \frac{\mu}{m_{3/2}} + 32\lambda \frac{\mu^2}{m_{3/2}^2} \right\}, \end{aligned} \quad (\text{C.21})$$

$$\begin{aligned} \langle a^0 | \mathbf{M}_{\mathbf{A}}^2 | t_I^0 \rangle &= \langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle + \frac{v}{2\sqrt{2}\Lambda} (1+\lambda) \text{Tr } \mathbf{M}_{\mathbf{A}(0)}^2 \\ &= \frac{v}{\sqrt{2}} \frac{m_{3/2}^2}{\Lambda} \left\{ \frac{9}{2} + \left(2\varsigma - \frac{1}{2} \right) \lambda - (2+\varsigma)\lambda^2 + [3 + (\lambda-2)\varsigma] \frac{\mu}{m_{3/2}} \right\}, \end{aligned} \quad (\text{C.22})$$

$$\begin{aligned} \langle t_I^0 | \mathbf{M}_{\mathbf{A}}^2 | t_I^0 \rangle &= \langle t_I^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle \left[1 - \frac{v^2}{4\Lambda^2} (1+\lambda)(1-3\lambda) \right] \\ &+ \frac{v}{\sqrt{2}\Lambda} (1+\lambda) \langle a^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle + \frac{v^2}{8\Lambda^2} (1+\lambda)^2 \langle t_I^0 | \mathbf{M}_{\mathbf{A}(0)}^2 | t_I^0 \rangle + \dots \\ &= 9m_{3/2}^2 + v^2 \frac{m_{3/2}^2}{8\Lambda^2} \left\{ -9 - 4(2\varsigma-5)\lambda + (7+8\varsigma)\lambda^2 \right. \\ &\left. - 2(2+\varsigma)\lambda^3 + 2 \left[-3 + (\lambda-2)^2\varsigma + 6\lambda \right] \frac{\mu}{m_{3/2}} \right\}. \end{aligned} \quad (\text{C.23})$$

This shows that, for μ and $m_{3/2}$ moderately larger than v , a^0 and t_I^0 are approximate

¹¹Since $\mathbf{M}_{\mathbf{A}(0)}^2$ is written in the basis $\{G^0, a^0, t_I^0\}$, eqs. (C.18)-(C.20), then \mathbf{N} in (C.9) has to be rotated to that same basis.

mass eigenstates up to $\mathcal{O}(v/\Lambda)$ corrections. Their masses can be read off directly from the formulae above.

The 2×2 mass matrix $\mathbf{M}_{\text{ch}(0)}^2$ for charged Higgses, in the basis $\{\phi_i^+\} = \{H_1^+, H_2^+\}$, is obtained from a formula similar to (C.1). By a suitable change of basis, the charged Goldstone boson $G^+ \equiv (H_1^+ - H_2^+)/\sqrt{2}$ can be exposed as an eigenstate of this matrix, with zero eigenvalue. The other eigenstate is the physical Higgs $H^+ \equiv (H_1^+ + H_2^+)/\sqrt{2}$ with mass given by the trace of $\mathbf{M}_{\text{ch}(0)}^2$. Using (C.2), it can be related to the mass of H^0 as

$$\langle H^- | \mathbf{M}_{\text{ch}(0)}^2 | H^+ \rangle = \langle H^0 | \mathbf{M}_{\text{h}(0)}^2 | H^0 \rangle - \frac{1}{4} g'^2 v^2 . \quad (\text{C.24})$$

Once again, to get the true mass we need to take into account that kinetic terms should be canonical. Up to $\mathcal{O}(v^2/\Lambda^2)$, the kinetic piece of the Lagrangian for charged Higgses reads

$$\delta \mathcal{L}_{\text{kin}} = \left[\partial_\mu H_1^- \left(\partial^\mu H_1^- \right)^* + \partial_\mu H_2^+ \left(\partial^\mu H_2^+ \right)^* \right] \left[1 + \frac{v^2}{2\Lambda^2} (1 + \lambda) \right] . \quad (\text{C.25})$$

This is canonical again after the re-scaling

$$\begin{bmatrix} H_1^+ \\ H_2^+ \end{bmatrix} \rightarrow \mathbf{N}' \begin{bmatrix} H_1^+ \\ H_2^+ \end{bmatrix} , \quad (\text{C.26})$$

with

$$\mathbf{N}' \equiv \left[1 - \frac{v^2}{4\Lambda^2} (1 + \lambda) \right] \mathbf{I}_2 . \quad (\text{C.27})$$

The corrected mass for the charged Higgs boson is therefore

$$M_{H^+}^2 \equiv \langle H^- | \mathbf{M}_{\text{ch}(0)}^2 | H^+ \rangle \left[1 - \frac{v^2}{2\Lambda^2} (1 + \lambda) \right] = M_{H^0}^2 - \frac{1}{4} g'^2 v^2 , \quad (\text{C.28})$$

with $M_{H^0}^2$ as defined in eq. (C.12).

References

- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].
- [2] V. A. Rubakov, hep-ph/0104152, and references therein.
- [3] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, Phys. Rev. D **59** (1999) 086001, [hep-th/9803235];
R. Altendorfer, J. Bagger and D. Nemeschansky, Phys. Rev. D **63** (2001) 125025,

[hep-th/0003117];

T. Gherghetta and A. Pomarol, Nucl. Phys. B **586** (2000) 141, [hep-ph/0003129];

A. Falkowski, Z. Lalak and S. Pokorski, Phys. Lett. B **491** (2000) 172, [hep-th/0004093];

P. Brax, A. Falkowski and Z. Lalak, hep-th/0107257.

[4] M. A. Luty and R. Sundrum, Phys. Rev. D **64** (2001) 065012 [hep-th/0012158].

[5] J. Bagger, D. Nemeschansky and R. Zhang, hep-th/0012163.

[6] A. Falkowski, Z. Lalak and S. Pokorski, hep-th/0102145.

[7] K. Behrndt and M. Cvetič, Phys. Lett. B **475** (2000) 253 [hep-th/9909058]; Phys. Rev. D **61** (2000) 101901 [hep-th/0001159];

R. Kallosh, A. D. Linde and M. Shmakova, JHEP **9911** (1999) 010 [hep-th/9910021];

R. Kallosh and A. D. Linde, JHEP **0002** (2000) 005 [hep-th/0001071];

M. Cvetič, H. Lu and C. N. Pope, hep-th/0002054;

N. Alonso-Alberca, P. Meessen and T. Ortín, Phys. Lett. B **482** (2000) 400 [hep-th/0003248];

H. Verlinde, hep-th/0004003;

K. Behrndt, Phys. Lett. B **487** (2000) 30 [hep-th/0005185];

P. Mayr, JHEP **0011** (2000) 013 [hep-th/0006204];

E. Bergshoeff, R. Kallosh and A. Van Proeyen, JHEP **0010** (2000) 033 [hep-th/0007044];

M. J. Duff, J. T. Liu and K. S. Stelle, hep-th/0007120;

K. Behrndt, C. Herrmann, J. Louis and S. Thomas, JHEP **0101** (2001) 011 [hep-th/0008112];

M. Zucker, Phys. Rev. D **64** (2001) 024024 [hep-th/0009083];

A. Falkowski, Z. Lalak and S. Pokorski, Phys. Lett. B **509** (2001) 337 [hep-th/0009167];

M. Cvetič, M. J. Duff, J. T. Liu, H. Lu, C. N. Pope and K. S. Stelle, Nucl. Phys. B **605** (2001) 141 [hep-th/0011167];

T. Gherghetta and A. Pomarol, Nucl. Phys. B **602** (2001) 3 [hep-ph/0012378];

S. Nojiri and S. D. Odintsov, Phys. Rev. D **64** (2001) 023502 [hep-th/0102032];

- M. A. Luty and R. Sundrum, hep-th/0105137;
D. Martí and A. Pomarol, hep-th/0106256.
- [8] J. E. Kim and H. P. Nilles, Phys. Lett. B **138** (1984) 150.
- [9] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B **212** (1983) 413.
- [10] J. Bagger and J. Wess, “Supersymmetry And Supergravity,” *Princeton, USA: Univ. Pr. (1992) 259 p.*
- [11] S. H. Shenker, RU-90-47 *Presented at the Cargese Workshop on Random Surfaces, Quantum Gravity and Strings, Cargese, France, May 28 - Jun 1, 1990*;
T. Banks and M. Dine, Phys. Rev. D **50** (1994) 7454 [hep-th/9406132];
J. A. Casas, Nucl. Phys. Proc. Suppl. **52A** (1997) 289 [hep-th/9608010].
- [12] L. J. Hall, J. Lykken and S. Weinberg, Phys. Rev. D **27** (1983) 2359;
S. K. Soni and H. A. Weldon, Phys. Lett. B **126**, 215 (1983);
V. S. Kaplunovsky and J. Louis, Phys. Lett. B **306** (1993) 269 [hep-th/9303040].
- [13] M. Cvetič, L. L. Everett and J. Wang, Nucl. Phys. B **538** (1999) 52 [hep-ph/9807321];
I. L. Buchbinder, M. Cvetič and A. Y. Petrov, Nucl. Phys. B **571** (2000) 358 [hep-th/9906141].
- [14] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. **84** (2000) 2080 [hep-ph/9909255].
- [15] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39** (1989) 844.
- [16] N. Polonsky and S. Su, Phys. Lett. B **508** (2001) 103 [hep-ph/0010113].
- [17] S. P. Martin, Phys. Rev. D **61** (2000) 035004 [hep-ph/9907550].
- [18] D. E. Groom *et al.* [Particle Data Group Collab.], Eur. Phys. J. C **15** (2000) 1.
- [19] S. Weinberg, Phys. Rev. D **19** (1979) 1277;
L. Susskind, Phys. Rev. D **20** (1979) 2619;

- P. Sikivie, L. Susskind, M. Voloshin and V. Zakharov, Nucl. Phys. B **173** (1980) 189.
- [20] S. Weinberg, “The quantum theory of fields. Vol. 3: Supersymmetry,” *Cambridge, UK: Univ. Pr. (2000) 419 p.*
- [21] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133** (1983) 61;
A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145** (1987) 1, and references therein.
- [22] E. Witten, Phys. Lett. B **155** (1985) 151.
- [23] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, Phys. Lett. B **131** (1983) 59;
B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B **318** (1993) 447 [hep-ph/9308325];
T. Banks, D. B. Kaplan and A. E. Nelson, Phys. Rev. D **49** (1994) 779 [hep-ph/9308292].
- [24] Z. Lalak, hep-th/0109074.
- [25] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480;
J. A. Casas and C. Muñoz, Phys. Lett. B **306** (1993) 288 [hep-ph/9302227].
- [26] P. Horava and E. Witten, Nucl. Phys. B **460** (1996) 506 [hep-th/9510209]; Nucl. Phys. B **475** (1996) 94 [hep-th/9603142].
- [27] L. E. Ibañez, C. Muñoz and S. Rigolin, Nucl. Phys. B **553** (1999) 43 [hep-ph/9812397].
- [28] LEP Collaborations and the LEP Higgs Working Group, LHWG Note 2001-03, ALEPH 2001-066 CONF 2001-046, DELPHI 2001-113 CONF 536, L3 Note 2699, OPAL Physics Note PN479.
- [29] LEP Collaborations and the LEP Higgs Working Group, LHWG Note 2001-04, ALEPH 2001-057 CONF 2001-037, DELPHI 2001-114 CONF 537, L3 Note 2700, OPAL Technical Note TN699.

- [30] P. Langacker and H. A. Weldon, Phys. Rev. Lett. **52** (1984) 1377;
H. A. Weldon, Phys. Lett. B **146** (1984) 59;
D. Comelli and J. R. Espinosa, Phys. Lett. B **388** (1996) 793 [hep-ph/9607400].
- [31] K. Riesselmann and S. Willenbrock, Phys. Rev. D **55** (1997) 311 [hep-ph/9608280];
T. Hambye and K. Riesselmann, Phys. Rev. D **55** (1997) 7255 [hep-ph/9610272].
- [32] J. R. Espinosa and M. Quirós, Phys. Rev. Lett. **81** (1998) 516 [hep-ph/9804235].
- [33] See *e.g.* J. R. Espinosa and R. Zhang, JHEP **0003** (2000) 026 [hep-ph/9912236];
Nucl. Phys. B **586** (2000) 3 [hep-ph/0003246];
M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. Wagner and G. Weiglein,
Nucl. Phys. B **580** (2000) 29 [hep-ph/0001002];
J. R. Espinosa and I. Navarro, hep-ph/0104047;
G. Degrandi, P. Slavich and F. Zwirner, hep-ph/0105096.
- [34] S. Y. Choi, J. Kalinowski, G. Moortgat-Pick and P. M. Zerwas, hep-ph/0108117.